

## Chapter 2

### Introduction to Engineering Calculations

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#### 2.1 Unit conversion

(a)

From Table A.9 (Appendix A):  $1 \text{ cP} = 10^{-3} \text{ kg m}^{-1} \text{ s}^{-1}$

$1 \text{ m} = 100 \text{ cm}$

Therefore:

$$1.5 \times 10^{-6} \text{ cP} = 1.5 \times 10^{-6} \text{ cP} \cdot \left| \frac{10^{-3} \text{ kg m}^{-1} \text{ s}^{-1}}{1 \text{ cP}} \right| \cdot \left| \frac{1 \text{ m}}{100 \text{ cm}} \right| = 1.5 \times 10^{-11} \text{ kg s}^{-1} \text{ cm}^{-1}$$

*Answer:*  $1.5 \times 10^{-11} \text{ kg s}^{-1} \text{ cm}^{-1}$

(b)

From Table A.8 (Appendix A):  $1 \text{ hp (British)} = 42.41 \text{ Btu min}^{-1}$

Therefore:

$$0.122 \text{ hp} = 0.122 \text{ hp} \cdot \left| \frac{42.41 \text{ Btu min}^{-1}}{1 \text{ hp}} \right| = 5.17 \text{ Btu min}^{-1}$$

*Answer:*  $5.17 \text{ Btu min}^{-1}$

(c)

$1 \text{ min} = 60 \text{ s}$

rpm means revolutions per minute. As revolutions is a non-dimensional quantity (Section 2.1.2), the units of rpm are  $\text{min}^{-1}$ . Therefore:

$$10,000 \text{ min}^{-1} = 10,000 \text{ min}^{-1} \cdot \left| \frac{1 \text{ min}}{60 \text{ s}} \right| = 167 \text{ s}^{-1}$$

*Answer:*  $167 \text{ s}^{-1}$

(d)

From Table A.8 (Appendix A):  $1 \text{ W} = 1 \text{ J s}^{-1}$

From Table A.7 (Appendix A):  $1 \text{ J} = 9.869 \times 10^{-3} \text{ l atm}$

From Table A.1 (Appendix A):  $1 \text{ m} = 3.281 \text{ ft}$

$1 \text{ min} = 60 \text{ s}$

As explained in Section 2.4.6,  $^{\circ}\text{C}^{-1}$  is the same as  $\text{K}^{-1}$ . Therefore:

$$\begin{aligned} 4335 \text{ W m}^{-2} \text{ }^{\circ}\text{C}^{-1} &= 4335 \text{ W m}^{-2} \text{ }^{\circ}\text{C}^{-1} \cdot \left| \frac{1 \text{ J s}^{-1}}{1 \text{ W}} \right| \cdot \left| \frac{9.869 \times 10^{-3} \text{ l atm}}{1 \text{ J}} \right| \cdot \left| \frac{60 \text{ s}}{1 \text{ min}} \right| \cdot \left| \frac{1 \text{ m}}{3.281 \text{ ft}} \right|^2 \\ &= 238.45 \text{ l atm min}^{-1} \text{ ft}^{-2} \text{ K}^{-1} \end{aligned}$$

*Answer:*  $238 \text{ l atm min}^{-1} \text{ ft}^{-2} \text{ K}^{-1}$

## 2.2 Unit conversion

(a)

From Table A.7 (Appendix A): 1 Btu = 0.2520 kcal

From Table A.3 (Appendix A): 1 lb = 453.6 g

Therefore:

$$345 \text{ Btu lb}^{-1} = 345 \text{ Btu lb}^{-1} \cdot \left| \frac{0.2520 \text{ kcal}}{1 \text{ Btu}} \right| \cdot \left| \frac{1 \text{ lb}}{453.6 \text{ g}} \right| = 0.192 \text{ kcal g}^{-1}$$

*Answer:* 0.192 kcal g<sup>-1</sup>

(b)

From Table A.5 (Appendix A): 1 mmHg = 1.316 × 10<sup>-3</sup> atm

From Table A.1 (Appendix A): 1 ft = 0.3048 m

From Table A.7 (Appendix A): 1 l atm = 9.604 × 10<sup>-2</sup> Btu

From Table A.8 (Appendix A): 1 Btu min<sup>-1</sup> = 2.391 × 10<sup>-2</sup> metric horsepower

1 m = 100 cm

1 l = 1000 cm<sup>3</sup>

1 h = 60 min

Therefore:

$$\begin{aligned} 670 \text{ mmHg ft}^3 &= 670 \text{ mmHg ft}^3 \cdot \left| \frac{1.316 \times 10^{-3} \text{ atm}}{1 \text{ mmHg}} \right| \cdot \left| \frac{9.604 \times 10^{-2} \text{ Btu}}{1 \text{ l atm}} \right| \cdot \left| \frac{0.3048 \text{ m}}{1 \text{ ft}} \right|^3 \cdot \left| \frac{100 \text{ cm}}{1 \text{ m}} \right|^3 \\ &\quad \cdot \left| \frac{1 \text{ l}}{1000 \text{ cm}^3} \right| \cdot \left| \frac{2.391 \times 10^{-2} \text{ metric horsepower}}{1 \text{ Btu min}^{-1}} \right| \cdot \left| \frac{1 \text{ h}}{60 \text{ min}} \right| \\ &= 9.56 \times 10^{-4} \text{ metric horsepower h} \end{aligned}$$

*Answer:* 9.56 × 10<sup>-4</sup> metric horsepower h

(c)

From Table A.7 (Appendix A): 1 kcal = 4.187 × 10<sup>3</sup> J

1 kcal = 1000 cal

1 kg = 1000 g

As explained in Section 2.4.6, °C<sup>-1</sup> is the same as K<sup>-1</sup>. Therefore:

$$\begin{aligned} 0.554 \text{ cal g}^{-1} \text{ °C}^{-1} &= 0.554 \text{ cal g}^{-1} \text{ °C}^{-1} \cdot \left| \frac{1 \text{ kcal}}{1000 \text{ cal}} \right| \cdot \left| \frac{4.187 \times 10^3 \text{ J}}{1 \text{ kcal}} \right| \cdot \left| \frac{1000 \text{ g}}{1 \text{ kg}} \right| \\ &= 2319.6 \text{ J kg}^{-1} \text{ K}^{-1} \end{aligned}$$

*Answer:* 2320 J kg<sup>-1</sup> K<sup>-1</sup>

(d)

From Table A.2 (Appendix A): 1 m<sup>3</sup> = 10<sup>3</sup> l

$$1 \text{ kg} = 1000 \text{ g}$$

Therefore:

$$10^3 \text{ g l}^{-1} = 10^3 \text{ g l}^{-1} \cdot \left| \frac{1 \text{ kg}}{1000 \text{ g}} \right| \cdot \left| \frac{10^3 \text{ l}}{1 \text{ m}^3} \right| = 10^3 \text{ kg m}^{-3}$$

*Answer:*  $10^3 \text{ kg m}^{-3}$

### 2.3 Unit conversion

(a)

From Table A.2 (Appendix A):  $1 \text{ m}^3 = 10^3 \text{ l}$

$$1 \text{ g} = 10^6 \mu\text{g}$$

$$1 \text{ l} = 1000 \text{ ml}$$

Therefore:

$$10^6 \mu\text{g ml}^{-1} = 10^6 \mu\text{g ml}^{-1} \cdot \left| \frac{1 \text{ g}}{10^6 \mu\text{g}} \right| \cdot \left| \frac{1000 \text{ ml}}{1 \text{ l}} \right| \cdot \left| \frac{10^3 \text{ l}}{1 \text{ m}^3} \right| = 10^6 \text{ g m}^{-3}$$

*Answer:*  $10^6 \text{ g m}^{-3}$

(b)

From Table A.9 (Appendix A):  $1 \text{ cP} = 10^{-3} \text{ Pa s}$

$$1 \text{ Pa s} = 1000 \text{ mPa s}$$

Therefore:

$$3.2 \text{ cP} = 3.2 \text{ cP} \cdot \left| \frac{10^{-3} \text{ Pa s}}{1 \text{ cP}} \right| \cdot \left| \frac{1000 \text{ mPa s}}{1 \text{ Pa s}} \right| = 3.2 \text{ mPa s}$$

*Answer:*  $3.2 \text{ mPa s}$

(c)

From Table A.7 (Appendix A):  $1 \text{ Btu} = 1.055 \times 10^3 \text{ J}$

From Table A.8 (Appendix A):  $1 \text{ W} = 1 \text{ J s}^{-1}$

From Table A.1 (Appendix A):  $1 \text{ ft} = 0.3048 \text{ m}$

$$1 \text{ h} = 3600 \text{ s}$$

From Section 2.4.6, a temperature difference of  $1 \text{ K}$  corresponds to a temperature difference of  $1.8 \text{ }^\circ\text{F}$ .

Therefore:

$$\begin{aligned} 150 \text{ Btu h}^{-1} \text{ ft}^{-2} (\text{ }^\circ\text{F ft}^{-1})^{-1} &= 150 \text{ Btu h}^{-1} \text{ ft}^{-1} \text{ }^\circ\text{F}^{-1} \cdot \left| \frac{1.055 \times 10^3 \text{ J}}{1 \text{ Btu}} \right| \cdot \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| \cdot \left| \frac{1 \text{ ft}}{0.3048 \text{ m}} \right| \\ &\quad \cdot \left| \frac{1.8 \text{ }^\circ\text{F}}{1 \text{ K}} \right| \cdot \left| \frac{1 \text{ W}}{1 \text{ J s}^{-1}} \right| \\ &= 259.6 \text{ W m}^{-1} \text{ K}^{-1} \end{aligned}$$

*Answer:*  $260 \text{ W m}^{-1} \text{ K}^{-1}$

(d)

$$1 \text{ h} = 3600 \text{ s}$$

rph means revolutions per hour. As revolutions is a non-dimensional quantity (Section 2.1.2), the units of rph are  $\text{h}^{-1}$ . Therefore:

$$66 \text{ rph} = 66 \text{ h}^{-1} \cdot \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| = 1.83 \times 10^{-2} \text{ s}^{-1}$$

Answer:  $1.83 \times 10^{-2} \text{ s}^{-1}$

## 2.4 Unit conversion and calculation

Convert to units of kg, m, s.

$$\text{From Table A.2 (Appendix A): } 1 \text{ ft}^3 = 2.832 \times 10^{-2} \text{ m}^3$$

$$\text{From Table A.2 (Appendix A): } 1 \text{ m}^3 = 10^3 \text{ l}$$

$$\text{From Table A.3 (Appendix A): } 1 \text{ lb} = 0.4536 \text{ kg}$$

$$\text{From Table A.8 (Appendix A): } 1 \text{ metric horsepower} = 7.355 \times 10^2 \text{ kg m}^2 \text{ s}^{-3}$$

$$\text{From Table A.1 (Appendix A): } 1 \text{ m} = 39.37 \text{ in.}$$

Therefore:

$$\begin{aligned}
 t_m &= 5.9 (2.3 \text{ m})^{2/3} \left( \frac{65 \text{ lb}}{\text{ft}^3} \frac{10,000 \text{ l} \cdot \left| \frac{1 \text{ m}^3}{10^3 \text{ l}} \right|}{0.70 \text{ metric hp} \cdot \left| \frac{7.355 \times 10^2 \text{ kg m}^2 \text{ s}^{-3}}{1 \text{ metric hp}} \right|} \cdot \left| \frac{1 \text{ ft}^3}{2.832 \times 10^{-2} \text{ m}^3} \right| \cdot \left| \frac{0.4536 \text{ kg}}{1 \text{ lb}} \right| \right)^{1/3} \\
 &\quad \left( \frac{2.3 \text{ m}}{45 \text{ in.}} \cdot \left| \frac{39.37 \text{ in.}}{1 \text{ m}} \right| \right)^{1/3} \\
 &= 5.9 (2.3)^{2/3} \text{ m}^{2/3} (2.724 \text{ m}^{-2/3} \text{ s}) 1.262 \\
 &= 35.3 \text{ s}
 \end{aligned}$$

Answer: 35 s

## 2.5 Unit conversion and dimensionless numbers

Case 1

Convert to units of kg, m, s.

$$\text{From Table A.3 (Appendix A): } 1 \text{ lb} = 0.4536 \text{ kg}$$

$$\text{From Table A.2 (Appendix A): } 1 \text{ ft}^3 = 2.832 \times 10^{-2} \text{ m}^3$$

$$\text{From Table A.9 (Appendix A): } 1 \text{ cP} = 10^{-3} \text{ kg m}^{-1} \text{ s}^{-1}$$

$$1 \text{ m} = 100 \text{ cm} = 1000 \text{ mm}$$

Using Eq. (2.1):

$$Re = \frac{\left(2 \text{ mm} \cdot \left| \frac{1 \text{ m}}{1000 \text{ mm}} \right| \right) \left(3 \text{ cm s}^{-1} \cdot \left| \frac{1 \text{ m}}{100 \text{ cm}} \right| \right) \left(25 \text{ lb ft}^{-3} \cdot \left| \frac{0.4536 \text{ kg}}{1 \text{ lb}} \right| \cdot \left| \frac{1 \text{ ft}^3}{2.832 \times 10^{-2} \text{ m}^3} \right| \right)}{10^{-6} \text{ cP} \cdot \left| \frac{10^{-3} \text{ kg m}^{-1} \text{ s}^{-1}}{1 \text{ cP}} \right|}$$

$$= 2.4 \times 10^7$$

Answer:  $2.4 \times 10^7$

### Case 2

Convert to units of kg, m, s.

From Table A.1 (Appendix A):  $1 \text{ in.} = 2.54 \times 10^{-2} \text{ m}$

From Table A.9 (Appendix A):  $1 \text{ lb ft}^{-1} \text{ h}^{-1} = 4.134 \times 10^{-4} \text{ kg m}^{-1} \text{ s}^{-1}$

$1 \text{ h} = 3600 \text{ s}$

Using Eq. (2.1):

$$Re = \frac{\left(1 \text{ in.} \cdot \left| \frac{2.54 \times 10^{-2} \text{ m}}{1 \text{ in.}} \right| \right) (1 \text{ m s}^{-1}) (12.5 \text{ kg m}^{-3})}{0.14 \times 10^{-4} \text{ lb}_m \text{ s}^{-1} \text{ ft}^{-1} \cdot \left| \frac{4.134 \times 10^{-4} \text{ kg m}^{-1} \text{ s}^{-1}}{1 \text{ lb ft}^{-1} \text{ h}^{-1}} \right| \cdot \left| \frac{3600 \text{ s}}{\text{h}} \right|} = 1.5 \times 10^4$$

Answer:  $1.5 \times 10^4$

## 2.6 Property data

Values were obtained from *Perry's Chemical Engineers' Handbook*, 8th edition, McGraw-Hill. Other sources may also be used.

### (a)

The viscosity of ethanol can be evaluated as a function of temperature using the coefficients listed in Table 2-313 of *Perry's Chemical Engineers' Handbook*. Converting from °C to K using Eq. (2.27),  $T(\text{K}) = 40 + 273.15 = 313.15 \text{ K}$ . Using the equation provided, the viscosity at 40°C is calculated as  $8.18 \times 10^{-4} \text{ Pa s}$ . Alternatively, using the nomograph of Figure 2-32 in *Perry's Chemical Engineers' Handbook*, the viscosity of ethanol at 40°C is estimated at about 0.82 cP, which is consistent with the previous result.

Answer: 0.82 cP

### (b)

From Table 2-325 in *Perry's Chemical Engineers' Handbook*, at 25°C and 1 atm pressure, the diffusivity of oxygen in water is  $2.5 \times 10^{-5} \text{ cm}^2 \text{ s}^{-1}$ .

Answer:  $2.5 \times 10^{-5} \text{ cm}^2 \text{ s}^{-1}$

### (c)

The thermal conductivity of borosilicate-type glass is listed in Table 2-326 of *Perry's Chemical Engineers' Handbook*. For temperatures between 30°C and 75°C, the thermal conductivity is  $0.63 \text{ Btu h}^{-1} \text{ ft}^{-2} (\text{°F/ft})^{-1}$ .

Answer:  $0.63 \text{ Btu h}^{-1} \text{ ft}^{-2} (\text{°F/ft})^{-1}$

(d)

The density of 100% acetic acid at 20°C is listed in Table 2-109 of *Perry's Chemical Engineers' Handbook* as 1.0498 g cm<sup>-3</sup>.

Answer: 1.0498 g cm<sup>-3</sup>

(e)

The specific heat capacity of liquid water at 80°C can be calculated using the values listed in Table 2-305 of *Perry's Chemical Engineers' Handbook*. Converting from °C to K using Eq. (2.27),  $T(\text{K}) = 80 + 273.15 = 353.15$  K. From the table,  $C_p = 0.075567$  kJ mol<sup>-1</sup> K<sup>-1</sup> at 350 K and  $C_p = 0.075708$  kJ mol<sup>-1</sup> K<sup>-1</sup> at 360 K. Assuming that  $C_p$  changes linearly with temperature between 350 K and 360 K, the value at 353.15 K can be interpolated as:

$$\begin{aligned} C_p(353.15 \text{ K}) &= 0.075567 \text{ kJ mol}^{-1} \text{ K}^{-1} + \frac{(353.15 - 350) \text{ K}}{(360 - 350) \text{ K}} (0.075708 - 0.075567) \text{ kJ mol}^{-1} \text{ K}^{-1} \\ &= 0.075611 \text{ kJ mol}^{-1} \text{ K}^{-1} \end{aligned}$$

Answer: 0.07561 kJ mol<sup>-1</sup> K<sup>-1</sup>

## 2.7 Dimensionless groups and property data

From *Perry's Chemical Engineers' Handbook*, the diffusivity of oxygen in water at 25°C and 1 atm pressure is  $2.5 \times 10^{-5}$  cm<sup>2</sup> s<sup>-1</sup>. Assuming this is the same at 28°C,  $\mathcal{D} = 2.5 \times 10^{-5}$  cm<sup>2</sup> s<sup>-1</sup>. As fermentation medium is mostly water, it is reasonable to assume that the density of liquid in the fermenter is the same as that of water. Unless the culture produces a highly viscous extracellular product such as gum, it is also reasonable to assume that the viscosity of liquid in the fermenter is the same as that of water. From *Perry's Chemical Engineers' Handbook*, the density of water at 28°C  $\rho_L = 0.9962652$  g cm<sup>-3</sup> and the viscosity of water at 28°C  $\mu_L = 0.87$  cP. The density of oxygen at 28°C and 1 atm pressure can be calculated using the ideal gas law. As molar density is the same as  $n/V$ , from Eq. (2.35):

$$\rho_G = \frac{n}{V} = \frac{p}{RT}$$

Temperature in the ideal gas equation is absolute temperature; therefore, from Eq. (2.27):

$$T(\text{K}) = (28 + 273.15) \text{ K} = 301.15 \text{ K}$$

From Appendix B,  $R = 82.057$  cm<sup>3</sup> atm K<sup>-1</sup> gmol<sup>-1</sup>. Substituting parameter values into the above equation for gas density gives:

$$\rho_G = \frac{1 \text{ atm}}{(82.057 \text{ cm}^3 \text{ atm K}^{-1} \text{ gmol}^{-1})(301.15 \text{ K})} = 4.05 \times 10^{-5} \text{ gmol cm}^{-3}$$

Using the atomic weights in Table C.1 (Appendix C), the molecular weight of oxygen is 32.0. Converting the result for  $\rho_G$  to mass terms:

$$\rho_G = 4.05 \times 10^{-5} \text{ gmol cm}^{-3} \cdot \left| \frac{32.0 \text{ g}}{1 \text{ gmol}} \right| = 1.30 \times 10^{-3} \text{ g cm}^{-3}$$

From Table A.9 (Appendix A), 1 cP = 10<sup>-2</sup> g cm<sup>-1</sup> s<sup>-1</sup>. From Eq. (2.17),  $g = 980.66$  cm s<sup>-2</sup>. 1 cm = 10 mm. The parameter values and conversion factors can now be used to calculate the dimensionless groups in the equation for the Sherwood number.

$$Gr = \frac{(2 \text{ mm})^3 (1.30 \times 10^{-3} \text{ g cm}^{-3})(0.9962652 - 1.30 \times 10^{-3}) \text{ g cm}^{-3} (980.66 \text{ cm s}^{-2}) \cdot \left| \frac{1 \text{ cm}}{10 \text{ mm}} \right|^3}{(0.87 \text{ cP})^2 \cdot \left| \frac{10^{-2} \text{ g cm}^{-1} \text{ s}^{-1}}{1 \text{ cP}} \right|^2}$$

$$= 134$$

Similarly for the Schmidt number:

$$Sc = \frac{0.87 \text{ cP} \cdot \left| \frac{10^{-2} \text{ g cm}^{-1} \text{ s}^{-1}}{1 \text{ cP}} \right|}{(0.9962652 \text{ g cm}^{-3}) (2.5 \times 10^{-5} \text{ cm}^2 \text{ s}^{-1})} = 349$$

Therefore:

$$Sh = 0.31 (134)^{1/3} (349)^{1/3} = 11.2$$

From the equation for  $Sh$ :

$$k_L = \frac{Sh \mathcal{D}}{D_b} = \frac{(11.2)(2.5 \times 10^{-5} \text{ cm}^2 \text{ s}^{-1})}{2 \text{ mm} \cdot \left| \frac{1 \text{ cm}}{10 \text{ mm}} \right|} = 1.40 \times 10^{-3} \text{ cm s}^{-1}$$

*Answer:*  $1.40 \times 10^{-3} \text{ cm s}^{-1}$

## 2.8 Dimensionless numbers and dimensional homogeneity

First, evaluate the units of the groups  $(C_p \mu / k)$  and  $(DG/\mu)$ :

$$\text{Units of } \left( \frac{C_p \mu}{k} \right) = \frac{(\text{Btu lb}^{-1} \text{ }^\circ\text{F}^{-1}) \text{ lb h}^{-1} \text{ ft}^{-1}}{\text{Btu h}^{-1} \text{ ft}^{-2} (\text{ }^\circ\text{F ft}^{-1})^{-1}} = 1$$

$$\text{Units of } \left( \frac{DG}{\mu} \right) = \frac{(\text{ft}) \text{ lb h}^{-1} \text{ ft}^{-2}}{\text{lb h}^{-1} \text{ ft}^{-1}} = 1$$

Therefore, these groups are dimensionless. For the equation to be dimensionally homogeneous,  $(h/C_p G)$  must also be dimensionless; the units of  $h$  must therefore cancel the units of  $C_p G$ .

$$\text{Units of } h = \text{units of } C_p G = (\text{Btu lb}^{-1} \text{ }^\circ\text{F}^{-1}) (\text{lb h}^{-1} \text{ ft}^{-2}) = \text{Btu } ^\circ\text{F}^{-1} \text{ h}^{-1} \text{ ft}^{-2}$$

The dimensions of  $h$  can be deduced from its units. From Table A.7 (Appendix A), Btu is a unit of energy with dimensions  $= \text{L}^2 \text{MT}^{-2}$ .  $^\circ\text{F}$  is a unit of temperature which, from Table 2.1, has the dimensional symbol  $\Theta$ . h (hour) is a unit of time with dimension T; ft is a unit of length with dimension L. Therefore:

$$\text{Dimensions of } h = \text{L}^2 \text{MT}^{-2} \Theta^{-1} \text{T}^{-1} \text{L}^{-2} = \text{MT}^{-3} \Theta^{-1}$$

*Answer:* Units =  $\text{Btu } ^\circ\text{F}^{-1} \text{ h}^{-1} \text{ ft}^{-2}$ ; dimensions =  $\text{MT}^{-3} \Theta^{-1}$

## 2.9 Dimensional homogeneity

$\lambda$  has dimensions L.  $\varepsilon$  has units  $\text{W kg}^{-1}$ ; therefore, from Tables A.8 and A.3 in Appendix A, the dimensions of  $\varepsilon$  are  $\text{L}^2 \text{MT}^{-3} \text{M}^{-1} = \text{L}^2 \text{T}^{-3}$ . Substituting this information into the equation for  $\lambda$ , for dimensional homogeneity:

$$\text{L} = \left( \frac{(\text{dimensions of } \nu)^3}{\text{L}^2 \text{T}^{-3}} \right)^{1/4} = \frac{(\text{dimensions of } \nu)^{3/4}}{\text{L}^{1/2} \text{T}^{-3/4}}$$

Solving for the dimensions of  $v$ :

$$(\text{dimensions of } v)^{3/4} = L^{3/2}T^{-3/4}$$

or

$$\text{dimensions of } v = L^2T^{-1}$$

*Answer:*  $L^2T^{-1}$

## 2.10 Dimensional homogeneity and $g_c$

The dimensions of  $D_i$  (length) = L. From Table 2.2, the dimensions of  $P = L^2MT^{-3}$  and the dimensions of  $\rho = L^{-3}M$ . From Section 2.3, the dimensions of  $g = LT^{-2}$ . From Section 2.1.2, the dimensions of rotational speed  $N_i = T^{-1}$ . Therefore:

$$\text{Dimensions of } \frac{P g}{\rho N_i^3 D_i^5} = \frac{(L^2MT^{-3})(LT^{-2})}{(L^{-3}M)(T^{-1})^3 L^5} = LT^{-2}$$

As  $N_p$  is a dimensionless number, equation (i) is not dimensionally homogeneous and therefore cannot be correct. From Section 2.3, the dimensions of  $g_c = 1$ . Therefore:

$$\text{Dimensions of } \frac{P g_c}{\rho N_i^3 D_i^5} = \frac{(L^2MT^{-3})(1)}{(L^{-3}M)(T^{-1})^3 L^5} = 1$$

Equation (ii) is dimensionally homogeneous and therefore likely to be correct.

*Answer:* (ii)

## 2.11 Mass and weight

From the definition of density in Section 2.4.1, mass is equal to density multiplied by volume. Therefore:

$$\text{Mass of water} = (10 \text{ ft}^3)(62.4 \text{ lb}_m \text{ ft}^{-3}) = 624 \text{ lb}_m$$

From Section 2.3, weight is the force with which a body is attracted to the centre of the earth by gravity. According to Newton's law (Section 2.3), this force is the mass of the body multiplied by gravitational acceleration.

(a)

From Eq. (2.18), at sea level and  $45^\circ$  latitude, gravitational acceleration  $g = 32.174 \text{ ft s}^{-2}$ . Therefore:

$$\text{Weight} = 624 \text{ lb}_m (32.174 \text{ ft s}^{-2}) = 2.008 \times 10^4 \text{ lb}_m \text{ ft s}^{-2}$$

Converting these units to  $\text{lb}_f$  using Eq. (2.19):

$$\text{Weight} = 2.008 \times 10^4 \text{ lb}_m \text{ ft s}^{-2} \cdot \left| \frac{1 \text{ lb}_f}{32.174 \text{ lb}_m \text{ ft s}^{-2}} \right| = 624 \text{ lb}_f$$

*Answer:*  $624 \text{ lb}_f$ . When  $g = 32.174 \text{ ft s}^{-2}$ ,  $\text{lb}$  mass is equal to  $\text{lb}$  force.

(b)

From Table A.1 (Appendix A),  $1 \text{ m} = 3.281 \text{ ft}$ . Using the same procedure as in (a):

$$\text{Weight} = 624 \text{ lb}_m (9.76 \text{ m s}^{-2}) \cdot \left| \frac{3.281 \text{ ft}}{1 \text{ m}} \right| = 1.998 \times 10^4 \text{ lb}_m \text{ ft s}^{-2}$$

Converting to  $\text{lb}_f$  using Eq. (2.19):



$$\text{Weight} = 1.998 \times 10^4 \text{ lb}_m \text{ ft s}^{-2} \cdot \left| \frac{1 \text{ lb}_f}{32.174 \text{ lb}_m \text{ ft s}^{-2}} \right| = 621 \text{ lb}_f$$

Answer: 621 lb<sub>f</sub>

## 2.12 Molar units

From the atomic weights in Table C.1 (Appendix C), the molecular weight of NaOH is 40.0.

(a)

From Eq. (2.22):

$$\text{lb-moles NaOH} = \frac{20.0 \text{ lb}}{40.0 \text{ lb lbmol}^{-1}} = 0.50 \text{ lbmol}$$

Answer: 0.50 lbmol

(b)

From Table A.3 (Appendix A), 1 lb = 453.6 g. Therefore:

$$20.0 \text{ lb} = 20.0 \text{ lb} \cdot \left| \frac{453.6 \text{ g}}{1 \text{ lb}} \right| = 9072 \text{ g}$$

From Eq. (2.21):

$$\text{gram-moles NaOH} = \frac{9072 \text{ g}}{40.0 \text{ g gmol}^{-1}} = 227 \text{ gmol}$$

Answer: 227 gmol

(c)

From Section 2.4.4, 1 kmol = 1000 gmol. Therefore, from (b):

$$\text{kg-moles NaOH} = 227 \text{ gmol} \cdot \left| \frac{1 \text{ kmol}}{1000 \text{ gmol}} \right| = 0.227 \text{ kmol}$$

Answer: 0.227 kmol

## 2.13 Density and specific gravity

(a)

From Section 2.4.1, the density of water at 4°C is 1.0000 g cm<sup>-3</sup>. Therefore, from Section 2.4.2, for a substance with specific gravity 1.5129<sub>4°C</sub><sup>20°C</sup>, the density at 20°C is 1.5129 g cm<sup>-3</sup>.

(i)

$$1 \text{ kg} = 1000 \text{ g}$$

$$1 \text{ m} = 100 \text{ cm}$$

Therefore:

$$\text{Density} = 1.5129 \text{ g cm}^{-3} \cdot \left| \frac{1 \text{ kg}}{1000 \text{ g}} \right| \cdot \left| \frac{100 \text{ cm}}{1 \text{ m}} \right|^3 = 1512.9 \text{ kg m}^{-3}$$

Answer: 1512.9 kg m<sup>-3</sup>

**(ii)**

From the atomic weights in Table C.1 (Appendix C), the molecular weight of nitric acid ( $\text{HNO}_3$ ) is 63.0. If the density of  $\text{HNO}_3$  at  $20^\circ\text{C}$  is  $1.5129 \text{ g cm}^{-3}$ , in  $1 \text{ cm}^3 \text{ HNO}_3$ , from Eq. (2.21):

$$\text{gram-moles} = \frac{1.5129 \text{ g}}{63.0 \text{ g gmol}^{-1}} = 0.024 \text{ gmol}$$

Therefore, the molar density is  $0.024 \text{ gmol cm}^{-3}$ . From the definition of specific volume in Section 2.4.3:

$$\text{Molar specific volume} = \frac{1}{\text{molar density}} = \frac{1}{0.024 \text{ gmol cm}^{-3}} = 41.67 \text{ cm}^3 \text{ gmol}^{-1}$$

*Answer:*  $41.67 \text{ cm}^3 \text{ gmol}^{-1}$

**(b)****(i)**

Density is defined as mass per unit volume (Section 2.4.1). Therefore, the mass flow rate is equal to the volumetric flow rate multiplied by the density:

$$\text{Mass flow rate} = (50 \text{ cm}^3 \text{ min}^{-1})(1.6 \text{ g cm}^{-3}) = 80 \text{ g min}^{-1}$$

*Answer:*  $80 \text{ g min}^{-1}$

**(ii)**

From the atomic weights in Table C.1 (Appendix C), the molecular weight of carbon tetrachloride is 153.8. Using the mass flow rate from (a):

$$\text{Molar flow rate} = 80 \text{ g min}^{-1} \cdot \left| \frac{1 \text{ gmol}}{153.8 \text{ g}} \right| = 0.52 \text{ gmol min}^{-1}$$

*Answer:*  $0.52 \text{ gmol min}^{-1}$

## 2.14 Molecular weight

From Section 2.4.5, the composition of air is approximately 21% oxygen and 79% nitrogen. For gases at low pressures, this means 21 mol%  $\text{O}_2$  and 79 mol%  $\text{N}_2$ . Therefore, in 1 gmol of air, there are 0.21 gmol  $\text{O}_2$  and 0.79 gmol  $\text{N}_2$ . From the atomic weights in Table C.1 (Appendix C), the molecular weights of  $\text{O}_2$  and  $\text{N}_2$  are 32.0 and 28.0, respectively. The molecular weight of air is equal to the number of grams in 1 gmol of air:

$$1 \text{ gmol air} = 0.21 \text{ gmol O}_2 \cdot \left| \frac{32.0 \text{ g}}{1 \text{ gmol}} \right| + 0.79 \text{ gmol N}_2 \cdot \left| \frac{28.0 \text{ g}}{1 \text{ gmol}} \right| = 28.8 \text{ g}$$

*Answer:* 28.8

## 2.15 Mole fraction

The molecular weights are listed in Table C.7 (Appendix C): water 18.0; ethanol 46.1; methanol 32.0; glycerol 92.1; acetic acid 60.1; benzaldehyde 106.1. In 100 g solution, there are 30 g water, 25 g ethanol, 15 g methanol, 12 g glycerol, 10 g acetic acid, 8 g benzaldehyde, and no other components. Therefore:

$$\text{Moles water} = 30 \text{ g} \cdot \left| \frac{1 \text{ gmol}}{18.0 \text{ g}} \right| = 1.67 \text{ gmol}$$

$$\text{Moles ethanol} = 25 \text{ g} \cdot \left| \frac{1 \text{ gmol}}{46.1 \text{ g}} \right| = 0.54 \text{ gmol}$$

$$\text{Moles methanol} = 15 \text{ g} \cdot \left| \frac{1 \text{ gmol}}{32.0 \text{ g}} \right| = 0.47 \text{ gmol}$$

$$\text{Moles glycerol} = 12 \text{ g} \cdot \left| \frac{1 \text{ gmol}}{92.1 \text{ g}} \right| = 0.13 \text{ gmol}$$

$$\text{Moles acetic acid} = 10 \text{ g} \cdot \left| \frac{1 \text{ gmol}}{60.1 \text{ g}} \right| = 0.17 \text{ gmol}$$

$$\text{Moles benzaldehyde} = 8 \text{ g} \cdot \left| \frac{1 \text{ gmol}}{106.1 \text{ g}} \right| = 0.075 \text{ gmol}$$

The total number of moles is  $1.67 + 0.54 + 0.47 + 0.13 + 0.17 + 0.075 = 3.055$  gmol. From Eq. (2.23):

$$\text{Mole fraction water} = \frac{1.67}{3.055} = 0.55$$

$$\text{Mole fraction ethanol} = \frac{0.54}{3.055} = 0.18$$

$$\text{Mole fraction methanol} = \frac{0.47}{3.055} = 0.15$$

$$\text{Mole fraction glycerol} = \frac{0.13}{3.055} = 0.043$$

$$\text{Mole fraction acetic acid} = \frac{0.17}{3.055} = 0.056$$

$$\text{Mole fraction benzaldehyde} = \frac{0.075}{3.055} = 0.025$$

*Answer:* 0.55 water; 0.18 ethanol; 0.15 methanol; 0.043 glycerol; 0.056 acetic acid; 0.025 benzaldehyde

## 2.16 Solution preparation

A 6% w/v solution means 6 g of  $\text{MgSO}_4 \cdot 7\text{H}_2\text{O}$  in 100 ml of solution (Section 2.4.5). To make up 100 ml of solution, weigh out 6 g of  $\text{MgSO}_4 \cdot 7\text{H}_2\text{O}$  using the balance and place the solid in the measuring cylinder. Add water to dissolve the solid and make up to the 100 ml mark on the measuring cylinder.

## 2.17 Moles, molarity, and composition

(a)

From the atomic weights in Table C.1 (Appendix C), the molecular weight of  $\text{C}_{12}\text{H}_{22}\text{O}_4$  is 230. 1 kg = 1000 g. Therefore:

$$21.2 \text{ kg} = 21.2 \text{ kg} \cdot \left| \frac{1000 \text{ g}}{1 \text{ kg}} \right| \cdot \left| \frac{1 \text{ gmol}}{230 \text{ g}} \right| = 92.2 \text{ gmol}$$

*Answer:* 92.2 gmol

**(b)**

From the atomic weights in Table C.1 (Appendix C), the molecular weight of  $C_{12}H_{22}O_{11}$  is 342.  $1 \text{ kg} = 1000 \text{ g}$ . Therefore:

$$4.5 \text{ kg s}^{-1} = 4.5 \text{ kg} \cdot \left| \frac{1000 \text{ g}}{1 \text{ kg}} \right| \cdot \left| \frac{1 \text{ gmol}}{342 \text{ g}} \right| = 13.2 \text{ gmol s}^{-1}$$

$1 \text{ min} = 60 \text{ s}$ . The amount of sucrose transferred in 30 min is:

$$13.2 \text{ gmol s}^{-1} (30 \text{ min}) \cdot \left| \frac{60 \text{ s}}{1 \text{ min}} \right| = 2.38 \times 10^4 \text{ gmol}$$

*Answer:*  $2.38 \times 10^4 \text{ gmol}$

**(c)**

$75 \text{ mM}$  means  $75 \times 10^{-3} \text{ gmol}$  per litre of solution (Section 2.4.5).  $1 \text{ l} = 1000 \text{ ml}$ ; therefore, in 10 ml or 1/100 of a litre, a  $75 \text{ mM}$  solution contains  $(75 \times 10^{-3}/100) \text{ gmol} = 75 \times 10^{-5} \text{ gmol}$ . From the atomic weights in Table C.1 (Appendix C), the molecular weight of  $C_4H_6O_6 \cdot H_2O$  is 168. Therefore:

$$75 \times 10^{-5} \text{ gmol} = 75 \times 10^{-5} \text{ gmol} \cdot \left| \frac{168 \text{ g}}{1 \text{ gmol}} \right| = 0.126 \text{ g}$$

*Answer:*  $0.126 \text{ g}$

**(d)**

$60 \mu\text{M}$  salicylaldehyde means  $60 \times 10^{-6} \text{ gmol}$  per litre of solution (Section 2.4.5).  $1 \text{ l} = 1000 \text{ ml}$ ; therefore, in 250 ml or 1/4 of a litre, a  $60 \mu\text{M}$  solution contains  $(60/4 \times 10^{-6}) \text{ gmol} = 15 \times 10^{-6} \text{ gmol}$ . From the atomic weights in Table C.1 (Appendix C), the molecular weight of  $C_7H_6O_2$  is 122. Therefore:

$$15 \times 10^{-6} \text{ gmol} = 15 \times 10^{-6} \text{ gmol} \cdot \left| \frac{122 \text{ g}}{1 \text{ gmol}} \right| = 1.83 \times 10^{-3} \text{ g}$$

$330 \text{ ppm}$  dichloroacetic acid means  $330 \text{ g}$  per  $10^6 \text{ g}$  of solution (Section 2.4.5). Because the solution being considered contains only very dilute concentrations of components in water, we can assume that the density of the solution is the same as the density of water or  $1 \text{ g ml}^{-1}$  (Section 2.4.1). Therefore, the concentration of dichloroacetic acid is  $330 \text{ g}$  per  $10^6 \text{ ml}$  of solution, and the mass of dichloroacetic acid in 250 ml of solution is  $330 \text{ g} \times (250 \text{ ml}/10^6 \text{ ml}) = 0.083 \text{ g}$ .

*Answer:*  $1.83 \times 10^{-3} \text{ g}$  of salicylaldehyde and  $0.083 \text{ g}$  of dichloroacetic acid

## 2.18 Concentration

**(a)**

If the concentration of NaCl added is the same as that already in the tank, the concentration remains constant at  $25 \text{ mM NaCl}$ .

*Answer:*  $25 \text{ mM}$

**(b)**

$25 \text{ mM NaCl}$  means  $25 \times 10^{-3} \text{ gmol NaCl}$  per litre (Section 2.4.5). Therefore, in the original 1500 l, there are  $25 \times 10^{-3} \text{ gmol l}^{-1} \times 1500 \text{ l} = 37.5 \text{ gmol NaCl}$ . After addition of the water, there are  $37.5 \text{ gmol NaCl}$  in 4500 litres of solution. Therefore, the concentration is  $37.5 \text{ gmol}/4500 \text{ l} = 8.33 \times 10^{-3} \text{ gmol l}^{-1}$  or  $8.33 \text{ mM}$ .

*Answer:* 8.3 mM

(c)

From the calculation in (b), the tank originally contains 37.5 gmol of NaCl. Adding 500 l of 25 mM NaCl increases the amount of NaCl present by  $25 \times 10^{-3} \text{ gmol l}^{-1} \times 500 \text{ l} = 12.5 \text{ gmol}$ , making the total amount of NaCl equal to  $(37.5 \text{ gmol} + 12.5 \text{ gmol}) = 50 \text{ gmol}$ . After addition of 3000 l of water, these 50 gmol of NaCl are present in  $(1500 \text{ l} + 500 \text{ l} + 3000 \text{ l}) = 5000 \text{ l}$  of solution.

(i)

There are 50 gmol of NaCl present in 5000 l of solution. Therefore, the concentration is  $50 \text{ gmol}/5000 \text{ l} = 0.01 \text{ gmol l}^{-1}$ . As  $1 \text{ gmol l}^{-1}$  is the same as 1 M (Section 2.4.5), the concentration is 0.01 M.

*Answer:* 0.01 M

(ii)

% w/v means g per 100 ml of solution (Section 2.4.5) From the atomic weights in Table C.1 (Appendix C), the molecular weight of NaCl is 58.44. Therefore, the mass corresponding to 50 gmol of NaCl is:

$$50 \text{ gmol} = 50 \text{ gmol} \cdot \left| \frac{58.44 \text{ g}}{1 \text{ gmol}} \right| = 2922 \text{ g}$$

This mass of NaCl is present in 5000 l of solution. Therefore, the concentration is  $2922 \text{ g}/5000 \text{ l} = 0.584 \text{ g l}^{-1}$ .  $1 \text{ l} = 1000 \text{ ml}$ . In 100 ml or 1/10 of a litre of solution, the mass of NaCl is  $0.584 \text{ g}/10 = 0.0584 \text{ g}$ . Therefore, the concentration is 0.0584 g per 100 ml, or 0.0584% w/v.

*Answer:* 0.0584% w/v

(iii)

From (ii), the concentration of NaCl is  $0.584 \text{ g l}^{-1}$ .  $1 \text{ l} = 1000 \text{ cm}^3$ . Therefore:

$$0.584 \text{ g l}^{-1} = 0.584 \text{ g l}^{-1} \cdot \left| \frac{1 \text{ l}}{1000 \text{ cm}^3} \right| = 5.84 \times 10^{-4} \text{ g cm}^{-3}$$

*Answer:*  $5.84 \times 10^{-4} \text{ g cm}^{-3}$

(iv)

ppm means g per  $10^6 \text{ g}$  of solution (Section 2.4.5) Because the solution of NaCl is very dilute, we can assume that the density of the solution is the same as the density of water =  $1 \text{ g cm}^{-3}$  (Section 2.4.1). Therefore, the concentration of NaCl calculated in (iii),  $5.84 \times 10^{-4} \text{ g cm}^{-3}$ , can be expressed as  $5.84 \times 10^{-4} \text{ g per g}$  of solution. The mass of NaCl present in  $10^6 \text{ g}$  of solution is  $(5.84 \times 10^{-4} \text{ g g}^{-1}) \times 10^6 \text{ g} = 584 \text{ g}$ , so the concentration is 584 g per  $10^6 \text{ g}$  of solution, or 584 ppm.

*Answer:* 584 ppm, assuming that the density of the solution is equal to the density of water.

## 2.19 Gas composition

Gas compositions are expressed as volume % (Section 2.4.5). For relatively light gases such as oxygen, carbon dioxide, ammonia and nitrogen, we can assume that the ideal gas law is valid over the range of conditions applying to bioreactor operation (Section 2.5). This means that the relative partial volumes of the component gases will not change with temperature and pressure, so that the gas composition will be unaffected.

*Answer:* The composition is unaffected.

## 2.20 Specific gravity and composition

A pharmaceutical concentration of 38.6% w/w means 38.6 g in 100 g of solution (Section 2.4.5). If the specific gravity of the solution is 1.036, the density referenced against water at 4°C is  $1.036 \text{ g cm}^{-3}$  (Section 2.4.2).

(a)

From the definition of density as mass per unit volume (Section 2.4.1), the volume of 100 g of solution =  $100 \text{ g}/(1.036 \text{ g cm}^{-3}) = 96.53 \text{ cm}^3$ . Therefore, the concentration of pharmaceutical is 38.6 g per  $96.53 \text{ cm}^3$  of solution, or  $38.6 \text{ g}/(96.53 \text{ cm}^3) = 0.40 \text{ g cm}^{-3}$ .  $1 \text{ l} = 1000 \text{ cm}^3$  and  $1 \text{ kg} = 1000 \text{ g}$ . Converting units gives:

$$0.40 \text{ g cm}^{-3} = 0.40 \text{ g cm}^{-3} \cdot \left| \frac{1000 \text{ cm}^3}{1 \text{ l}} \right| \cdot \left| \frac{1 \text{ kg}}{1000 \text{ g}} \right| = 0.40 \text{ kg l}^{-1}$$

Answer:  $0.40 \text{ kg l}^{-1}$

(b)

The flow rate of pharmaceutical is found by multiplying the solution flow rate by the pharmaceutical concentration:

$$\text{Pharmaceutical flow rate} = 8.6 \text{ l min}^{-1} \times 0.40 \text{ kg l}^{-1} = 3.44 \text{ kg min}^{-1}$$

$1 \text{ kg} = 1000 \text{ g}$ . Converting to molar units using the pharmaceutical molecular weight:

$$\text{Pharmaceutical flow rate} = 3.44 \text{ kg min}^{-1} \cdot \left| \frac{1000 \text{ g}}{1 \text{ kg}} \right| \cdot \left| \frac{1 \text{ gmol}}{1421 \text{ g}} \right| = 2.42 \text{ gmol min}^{-1}$$

Answer:  $2.42 \text{ gmol min}^{-1}$

## 2.21 Temperature scales

From Eq. (2.30):

$$-40 = 1.8 T(^{\circ}\text{C}) + 32$$

$$T(^{\circ}\text{C}) = -40$$

From Eq. (2.28):

$$T(^{\circ}\text{R}) = -40 + 459.67$$

$$T(^{\circ}\text{R}) = 420$$

From Eq. (2.27) and the result for  $T(^{\circ}\text{C})$ :

$$T(\text{K}) = -40 + 273.15$$

$$T(\text{K}) = 233$$

Answer:  $40^{\circ}\text{C}$ ,  $420^{\circ}\text{R}$ ,  $233 \text{ K}$

## 2.22 Pressure scales

(a)

Assume that the atmospheric (barometric) pressure is 14.7 psi (Section 2.4.7). From Eq. (2.31):

$$\text{Absolute pressure} = 15 \text{ psi} + 14.7 \text{ psi} = 29.7 \text{ psi}$$

From Table A.5 (Appendix A),  $1 \text{ psi} = 6.805 \times 10^{-2} \text{ atm}$ . Therefore:

$$\text{Absolute pressure} = 29.7 \text{ psi} \cdot \left| \frac{6.805 \times 10^{-2} \text{ atm}}{1 \text{ psi}} \right| = 2.02 \text{ atm}$$

*Answer:* 29.7 psi, 2.02 atm

**(b)**

Vacuum pressure is pressure below barometric (Section 2.4.7). If the barometric pressure is 14.7 psi (Section 2.4.7):

$$\text{Absolute pressure} = 14.7 \text{ psi} - 3 \text{ psi} = 11.7 \text{ psi}$$

*Answer:* 11.7 psi

### 2.23 Gas leak

The volume of the cylinder does not change as a result of the leak, so  $V_1 = V_2 = 48 \text{ l}$ .  $P_1 = 0.35 \text{ MPa} = 0.35 \times 10^6 \text{ Pa}$ . When the cylinder is left open to the atmosphere,  $P_2 = 1 \text{ atm}$ .  $T_1 = 22^\circ\text{C}$ ;  $T_2 = 33^\circ\text{C}$ . Let us assume that the ideal gas law applies to compressed air under the prevailing conditions (Section 2.5). Temperature in the ideal gas equation is absolute temperature; therefore, using Eq. (2.27),  $T_1 = (22 + 273.15) \text{ K} = 295.15 \text{ K}$  and  $T_2 = (33 + 273.15) \text{ K} = 306.15 \text{ K}$ . The ratio of the amounts of air in the tank before and after the leak can be determined using Eq. (2.35):

$$\frac{n_1}{n_2} = \frac{\frac{p_1 V_1}{RT_1}}{\frac{p_2 V_2}{RT_2}}$$

Because  $V_1 = V_2$ , these terms and  $R$  can be cancelled to give:

$$\frac{n_1}{n_2} = \frac{p_1 T_2}{p_2 T_1}$$

From Table A.5 (Appendix A),  $1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$ . Substituting values into the equation:

$$\frac{n_1}{n_2} = \frac{p_1 T_2}{p_2 T_1} = \frac{0.35 \times 10^6 \text{ Pa} \cdot \left| \frac{1 \text{ atm}}{1.013 \times 10^5 \text{ Pa}} \right| \cdot (306.15 \text{ K})}{1 \text{ atm} (295.15 \text{ K})} = 3.58$$

or

$$n_2 = 0.28 n_1$$

The amount of air in the tank after the leak is 28% of that present before the leak. The amount lost is therefore 72%.

*Answer:* 72%, assuming that the ideal gas law applies

### 2.24 Gas supply

The flow rate of air required to operate the bioreactor is  $0.8 \times 1.5 = 1.2 \text{ l min}^{-1}$ . The amount of air provided to the bioreactor between 4 pm Friday and 9 am Monday, i.e. over a period of 65 hours, can be determined using the ideal gas law with  $T = 25^\circ\text{C}$  and  $P = 1 \text{ atm}$ . Temperature in the ideal gas equation is absolute temperature; therefore, using Eq. (2.27),  $T = (25 + 273.15) \text{ K} = 298.15 \text{ K}$ . From Appendix B,  $R = 0.082057 \text{ l atm gmol}^{-1} \text{ K}^{-1}$ . Using Eq. (2.35), the amount of air required to operate the bioreactor is:

$$n = \frac{pV}{RT} = \frac{1 \text{ atm} (1.2 \text{ l min}^{-1}) (65 \text{ h}) \cdot \left| \frac{60 \text{ min}}{1 \text{ h}} \right|}{0.082057 \text{ l atm gmol}^{-1} \text{ K}^{-1} (298.15 \text{ K})} = 191 \text{ gmol}$$

This amount of air can be compared with the amount available in the gas cylinder.  $V = 48 \text{ l}$ .  $T = 20^\circ\text{C}$ ; using Eq. (2.27),  $T = (20 + 273.15) \text{ K} = 293.15 \text{ K}$ . If the gauge pressure is 800 psi, the absolute pressure  $P$  in the cylinder is  $(800 + 14.7) \text{ psi} = 814.7 \text{ psi}$  (Section 2.4.7). From Table A.5 (Appendix A),  $1 \text{ psi} = 6.805 \times 10^{-2} \text{ atm}$ . Applying Eq. (2.35):

$$n = \frac{pV}{RT} = \frac{814.7 \text{ psi} \cdot \left| \frac{6.805 \times 10^{-2} \text{ atm}}{1 \text{ psi}} \right| \cdot (48 \text{ l})}{0.082057 \text{ l atm gmol}^{-1} \text{ K}^{-1} (293.15 \text{ K})} = 111 \text{ gmol}$$

Even if the uncertainty factor of 5% is taken into account with respect to the values of  $n$  calculated, the amount of air in the cylinder is less than that required to operate the bioreactor over the weekend.

*Answer:* No

## 2.25 Stoichiometry and incomplete reaction

(a)

The molecular weights are calculated from Table C.1 (Appendix C): penicillin = 334.4; glucose = 180.2. The maximum theoretical yield from the stoichiometric equation is 1 gmol of penicillin for every 1.67 gmol of glucose. This is equivalent to 334.4 g of penicillin per  $1.67 \times 180.2 = 300.9 \text{ g}$  of glucose, or  $334.4 \text{ g}/300.9 \text{ g} = 1.1 \text{ g g}^{-1}$ .

*Answer:*  $1.1 \text{ g g}^{-1}$

(b)

The maximum theoretical yield in (a) is obtained when all the glucose consumed is directed into penicillin production according to the stoichiometric equation. If only 6% of the glucose is used in this way, the amount of penicillin produced for every 300.9 g of glucose consumed is much lower at  $334.4 \text{ g} \times 0.06 = 20.06 \text{ g}$ . Therefore, the actual yield of penicillin from glucose is  $20.06 \text{ g}/300.9 \text{ g} = 0.067 \text{ g g}^{-1}$ .

*Answer:*  $0.067 \text{ g g}^{-1}$

(c)

From the atomic weights in Table C.1 (Appendix C), the molecular weight of phenylacetic acid is 136.2.

(i)

The only possible limiting substrates are glucose and phenylacetic acid. Using a basis of 1 l of medium, if  $(50 - 5.5) = 44.5 \text{ g}$  of glucose are consumed but only 6% is available for penicillin synthesis, the mass of glucose used in the penicillin reaction is  $44.5 \times 0.06 = 2.67 \text{ g}$ . Converting to gmol of glucose, this is equivalent to  $2.67 \text{ g}/(180.2 \text{ g gmol}^{-1}) = 1.48 \times 10^{-2} \text{ gmol}$  of glucose available for penicillin synthesis. At the same time,  $4 \text{ g}$  or  $4 \text{ g}/(136.2 \text{ g gmol}^{-1}) = 2.94 \times 10^{-2} \text{ gmol}$  of phenylacetic acid is available which, according to the stoichiometric equation, requires  $1.67 \times (2.94 \times 10^{-2}) = 4.91 \times 10^{-2} \text{ gmol}$  of glucose for complete reaction. As the amount of glucose ( $4.91 \times 10^{-2} \text{ gmol}$ ) required for complete reaction of the phenylacetic acid is greater than the amount of glucose ( $1.48 \times 10^{-2} \text{ gmol}$ ) available after growth and maintenance activities, glucose is the limiting substrate.

*Answer:* Glucose



**(ii)**

Of the  $44.5 \text{ g l}^{-1}$  glucose consumed, 24% or  $10.7 \text{ g l}^{-1}$  is used for growth. In a 100-litre tank, the total mass of glucose consumed for growth is therefore  $10.7 \text{ g l}^{-1} \times 100 \text{ l} = 1070 \text{ g}$ , or 1.07 kg.

*Answer:* 1.07 kg

**(iii)**

From **(i)**,  $1.48 \times 10^{-2} \text{ gmol}$  of glucose is used in the penicillin reaction per litre. According to the stoichiometry, this produces  $(1.48 \times 10^{-2})/1.67 = 8.86 \times 10^{-3} \text{ gmol}$  of penicillin per litre. Therefore, in a 100-litre tank,  $(8.86 \times 10^{-3} \text{ gmol l}^{-1}) \times 100 \text{ l} = 0.886 \text{ gmol}$  of penicillin is formed. Converting to mass,  $0.886 \text{ gmol} \times 334.4 \text{ g gmol}^{-1} = 296 \text{ g}$  penicillin are formed.

*Answer:* 296 g

**(iv)**

If, from **(i)**,  $1.48 \times 10^{-2} \text{ gmol}$  glucose is used in the penicillin reaction per litre, from stoichiometry  $(1.48 \times 10^{-2})/1.67 = 8.86 \times 10^{-3} \text{ gmol l}^{-1}$  phenylacetic acid must also be used. Converting from moles to mass, this is equivalent to  $8.86 \times 10^{-3} \text{ gmol l}^{-1} \times 136.2 \text{ g gmol}^{-1} = 1.21 \text{ g l}^{-1}$  phenylacetic acid. As  $4 \text{ g l}^{-1}$  phenylacetic acid is provided,  $(4 - 1.21) \text{ g l}^{-1} = 2.79 \text{ g l}^{-1}$  phenylacetic acid must remain.

*Answer:*  $2.79 \text{ g l}^{-1}$

## 2.26 Stoichiometry, yield, and the ideal gas law

**(a)**

Adding up the numbers of C, H, O and N atoms on both sides of the equation shows that the equation is balanced: C (16 = 16), H (38.3 = 38.3), O (32.6 = 32.6), N (1.42 = 1.42).

*Answer:* Yes

**(b)**

The molecular weights are calculated from Table C.1 (Appendix C): cells = 91.5; hexadecane = 226.4. From the stoichiometry, as 1 gmol of hexadecane is required to produce 1.65 gmol of cells, the maximum yield is  $1.65 \text{ gmol} \times 91.5 \text{ g gmol}^{-1} = 151 \text{ g}$  cells per 226.4 g hexadecane, or  $151 \text{ g}/226.4 \text{ g} = 0.67 \text{ g g}^{-1}$ .

*Answer:*  $0.67 \text{ g g}^{-1}$

**(c)**

From the atomic weights in Table C.1 (Appendix C), the molecular weight of oxygen is 32.0. From the stoichiometry, 16.28 gmol of oxygen is required to produce 1.65 gmol of cells which, from **(b)**, is equal to 151 g of cells. The maximum yield is therefore 151 g of cells per  $(16.28 \text{ gmol} \times 32.0 \text{ g gmol}^{-1}) = 521 \text{ g}$  oxygen, or  $151 \text{ g}/521 \text{ g} = 0.29 \text{ g g}^{-1}$ .

*Answer:*  $0.29 \text{ g g}^{-1}$

**(d)**

$2.5 \text{ kg} = 2500 \text{ g}$ . Converting to molar terms using the cell molecular weight from **(b)**,  $2500 \text{ g} = 2500 \text{ g}/(91.5 \text{ g gmol}^{-1}) = 27.3 \text{ gmol}$  cells. The minimum amounts of substrates are required when 100% of the hexadecane is converted according to the stoichiometric equation.

**(i)**

From the stoichiometry, production of 27.3 gmol of cells requires  $27.3/1.65 = 16.5$  gmol of hexadecane. Converting to mass terms using the molecular weight of hexadecane,  $16.5 \text{ gmol} = 16.5 \text{ gmol} \times 226.4 \text{ g gmol}^{-1} = 3736 \text{ g} = 3.74 \text{ kg}$  of hexadecane.

*Answer:* 3.74 kg

**(ii)**

From the answer in **(d)(i)**, the concentration of hexadecane required is 3.74 kg in  $3 \text{ m}^3$ , or  $1.25 \text{ kg m}^{-3}$ .

*Answer:*  $1.25 \text{ kg m}^{-3}$

**(iii)**

According to the stoichiometric equation, production of 27.3 gmol of cells requires  $27.3 \times 16.28/1.65 = 269.4$  gmol of oxygen. As air contains approximately 21 mol% oxygen (Section 2.4.5), the moles of air required is  $269.4/0.21 = 1282.9$  gmol. The corresponding volume of air is calculated using the ideal gas law. From Eq. (2.35):

$$V = \frac{nRT}{p}$$

$P = 1 \text{ atm}$ ;  $T = 20^\circ\text{C}$ . As temperature in the ideal gas equation is absolute temperature, from Eq. (2.27):

$$T = (20 + 273.15) \text{ K} = 293.15 \text{ K}$$

From Appendix B,  $R = 82.057 \text{ cm}^3 \text{ atm K}^{-1} \text{ gmol}^{-1}$ .  $1 \text{ m} = 100 \text{ cm}$ . Substituting these values into the equation for  $V$ :

$$V = \frac{(1282.9 \text{ gmol})(82.057 \text{ cm}^3 \text{ atm K}^{-1} \text{ gmol}^{-1})(293.15 \text{ K})}{1 \text{ atm}} \cdot \left| \frac{1 \text{ m}}{100 \text{ cm}} \right|^3 = 31 \text{ m}^3$$

*Answer:*  $31 \text{ m}^3$

## 2.27 Stoichiometry and the ideal gas law

**(a)**

Adding up the numbers of C, H, O and N atoms on both sides of the equation shows that the equation is balanced: C ( $6 = 6$ ), H ( $12.6 \approx 12.7$ ), O ( $13.3 = 13.3$ ), N ( $0.33 = 0.33$ )

*Answer:* Yes

**(b)**

From Table C.1 (Appendix C), the molecular weights are: glucose = 180.2;  $\text{HNO}_3 = 63.0$ ; biomass 28.3. Converting to molar units, the concentration of glucose is  $30 \text{ g l}^{-1}/(180.2 \text{ g gmol}^{-1}) = 0.166 \text{ gmol l}^{-1}$ . From the stoichiometry, this requires  $0.18 \times 0.166 \text{ gmol l}^{-1} = 0.030 \text{ gmol l}^{-1}$  of  $\text{HNO}_3$  for complete conversion.

*Answer:*  $0.030 \text{ gmol l}^{-1}$

**(c)**

From the glucose concentration calculated in **(b)**, the gmol of glucose present in 50 l is  $0.166 \text{ gmol l}^{-1} \times 50 \text{ l} = 8.30 \text{ gmol}$ . According to the stoichiometry, complete conversion of 8.30 gmol of glucose produces  $2.5 \times 8.30 \text{ gmol} = 20.75 \text{ gmol}$  of biomass. Converting to mass units using the molecular weight of the biomass,  $20.75 \text{ gmol} \times 28.3 \text{ g gmol}^{-1} = 587 \text{ g}$  of biomass are produced.

*Answer:* 587 g

**(d)**

According to the stoichiometric equation, complete conversion of 8.30 gmol of glucose requires  $3.4 \times 8.30 = 28.2$  gmol of oxygen. The volume of oxygen required can be calculated using the ideal gas law with  $P = 1$  atm. As temperature in the ideal gas equation is absolute temperature,  $20^\circ\text{C}$  is converted to K using Eq. (2.27):

$$T = (20 + 273.15) \text{ K} = 293.15 \text{ K}$$

From Appendix B,  $R = 0.000082057 \text{ m}^3 \text{ atm K}^{-1} \text{ gmol}^{-1}$ . Using Eq. (2.35), the volume of oxygen required is:

$$V = \frac{nRT}{p} = \frac{(28.2 \text{ gmol})(0.000082057 \text{ m}^3 \text{ atm K}^{-1} \text{ gmol}^{-1})(293.15 \text{ K})}{1 \text{ atm}} = 0.678 \text{ m}^3$$

Air contains approximately 21 volume % oxygen (Section 2.4.5). Therefore, the volume of air required is  $0.678 \text{ m}^3 / 0.21 = 3.2 \text{ m}^3$ .

*Answer:*  $3.2 \text{ m}^3$

## 2.28 Stoichiometry, yield, and limiting substrate

**(a)**

Adding up the numbers of C, H and O atoms and the charges on both sides of the equation shows that the equation is balanced: C ( $10 = 10$ ), H ( $20 = 20$ ), O ( $34 = 34$ ), charge ( $-8 = -8$ ).

*Answer:* Yes

**(b)**

From Table C.1 (Appendix C), the molecular weights are  $\text{N}_2 = 28.0$  and acetate ( $\approx$  acetic acid) = 60.1. According to the stoichiometry, the yield is 4 gmol of  $\text{N}_2$  for every 5 gmol of acetate consumed. Converting to mass units using the molecular weights, the yield is  $(4 \times 28.0 \text{ g N}_2) / (5 \times 60.1 \text{ g acetate}) = 0.37 \text{ g g}^{-1}$ .

*Answer:*  $0.37 \text{ g g}^{-1}$

**(c)**

Effectively,  $0.75 \times 6 \text{ mM} = 4.5 \text{ mM}$  of acetic acid and  $0.85 \times 7 \text{ mM} = 5.95 \text{ mM}$  of nitrate are available for the denitrification reaction. From the reaction stoichiometry, 8 mol of nitrate are required for every 5 mol of acetic acid consumed. Therefore, complete reaction of 4.5 mM acetic acid requires  $4.5 \text{ mM} \times 8/5 = 7.2 \text{ mM}$  nitrate. As only 5.95 mM nitrate is available, nitrate is the limiting substrate.

*Answer:* Nitrate

**(d)**

The conversion is based on the amount of limiting substrate available. From (c), nitrate is the limiting substrate and 5.95 mM of nitrate is available for the denitrification reaction. Therefore, in 5000 l, the total amount of nitrate converted in the reaction is  $5.95 \times 10^{-3} \text{ gmol l}^{-1} \times 5000 \text{ l} = 29.75 \text{ gmol}$ . According to the stoichiometry, for each 8 gmol of nitrate reacted, 4 gmol of  $\text{N}_2$  are formed. Therefore, conversion of 29.75 gmol of nitrate produces  $29.75 \times 4/8 = 14.88 \text{ gmol}$  of  $\text{N}_2$ . Converting to mass units using the molecular weight of  $\text{N}_2$ , the mass of  $\text{N}_2$  produced is  $14.88 \text{ gmol} \times 28.0 \text{ g gmol}^{-1} = 416.6 \text{ g}$ .

*Answer:* 417 g

### 2.29 Order-of-magnitude calculation

rpm means revolutions per minute. As revolutions is a non-dimensional quantity (Section 2.1.2), the units of rpm are  $\text{min}^{-1}$ . Therefore:

$$12,000 \text{ min}^{-1} = 12,000 \text{ min}^{-1} \cdot \left| \frac{1 \text{ min}}{60 \text{ s}} \right| = 200 \text{ s}^{-1}$$

$\pi = 3.14159$ . From Eq. (2.16),  $g = 9.8066 \text{ m s}^{-2}$ . The equation for  $\Sigma$  using the actual data is:

$$\Sigma = \frac{3.14159 (200 \text{ s}^{-1})^2 (1.25 \text{ m}) (0.37 \text{ m})^2}{2 (9.8066 \text{ m s}^{-2})}$$

Using approximate values, this becomes:

$$\Sigma = \frac{3 (200)^2 (1) (0.4)^2 \text{ m}^3 \text{ s}^{-2}}{2 (10) \text{ m s}^{-2}} = 960 \text{ m}^2$$

This result is closer to  $1000 \text{ m}^2$  than to  $100 \text{ m}^2$ .

*Answer:*  $1000 \text{ m}^2$

### 2.30 Order-of-magnitude calculation

$\pi = 3.14159$ . The equation for  $r_{As}^*$  using the actual data is:

$$r_{As}^* = \frac{4}{3} (3.14159) (3.2 \times 10^{-3})^3 \text{ m}^3 \frac{(0.12 \text{ gmol s}^{-1} \text{ m}^{-3}) (41 \text{ gmol m}^{-3})}{(0.8 + 41) \text{ gmol m}^{-3}}$$

Using approximate values, this becomes:

$$r_{As}^* = \frac{4}{3} (3) (30 \times 10^{-9}) \frac{(4)}{(40)} \text{ gmol s}^{-1} = 120 \times 10^{-10} \text{ gmol s}^{-1} = 1.2 \times 10^{-8} \text{ gmol s}^{-1}$$

This result indicates that the student reporting the value  $1.6 \times 10^{-8} \text{ gmol s}^{-1}$  is more likely to be correct.

*Answer:*  $1.6 \times 10^{-8} \text{ gmol s}^{-1}$