SOLUTIONS
Chapter 2

2.1. The velocity of a particle is given by **V** = $10t^2 - 40t - 100$, where **V** is in m/s and t is in s. When is its acceleration zero?

Find: When acceleration $a = 0$.

Known: Velocity **V** = $10t^2 - 40t - 100$.

Using the definition of acceleration,

$$a = \frac{dV}{dt} = \frac{d}{dt}(10t^2 - 40t - 100) = 20t - 40 = 0.$$
$$t = 2 \text{ s}$$

Acceleration is zero after a time of 2 seconds.

2.2. An object has acceleration $a = 9t^2$. At time $t = 0$, its position $x_0 = 10$ m and velocity $\mathbf{V}_0 = -5$ m/s. What are its position and velocity when $t = 1$ s?

<u>Find:</u> Position x and velocity $\mathbf{V}$ at time $t = 1$ s.

<u>Known:</u> Acceleration $a = 9t^2$, position at $t = 0$ of $x_0 = 10$ m, velocity at $t = 0$ of $\mathbf{V}_0 = -5$ m/s.

We can obtain velocity $\mathbf{V}$ by integrating acceleration:

$$\mathbf{V} = \int a dt = \int 9t^2 dt = 3t^3 + C_1,$$

then using the boundary conditions of $\mathbf{V}_0 = -5$ m/s at $t = 0$ s,

$$\mathbf{V} = \underbrace{3t^3}_{0} + C_1 = -5,$$

$$C_1 = -5.$$

$$\mathbf{V} = 3t^3 - 5 = -2 \text{ m/s at } t = 1 \text{ s}.$$

We can obtain position x in a similar manner:

$$x = \int \mathbf{V} dt = \int (3t^3 - 5) dt = \frac{3}{4}t^4 - 5t + C_2,$$

then using the boundary conditions of $x_0 = 10$ m at $t = 0$ s,

$$x = \underbrace{\frac{3}{4}t^4}_{0} - \underbrace{5t}_{0} + C_2 = 10,$$

$$C_2 = 10.$$

$$x = \frac{3}{4}t^4 - 5t + 10 = 5.75 \text{ m at } t = 1 \text{ s}.$$

After one second, the position of the object is 5.75 m and the velocity is –2 m/s.

2.3. The acceleration of a mass is given by $a = 4t - 10$ where a is in m/s^2 and t is in s. If the mass starts at $t = 0$ with displacement $x = 0$ and velocity $\mathbf{V} = 2$ m/s, find the displacement as a function of time.

Find: Displacement x as a function of time.

Known: Acceleration $a = 4t - 10$ m/s^2, displacement at $t = 0$ of $x_0 = 0$, velocity at $t = 0$ of $\mathbf{V}_0 = 2$ m/s.

We can obtain velocity $\mathbf{V}$ by integrating acceleration:

$$\mathbf{V} = \int a dt = \int (4t - 10) dt = 2t^2 - 10t + \mathbf{V}_0,$$

then applying the initial condition of $\mathbf{V}_0 = 2$ m/s when $t = 0$ we get

$$\mathbf{V} = 2t^2 - 10t + 2.$$

Applying the same process to find position from velocity,

$$x = \int \mathbf{V} dt = \int (2t^2 - 10t + 2) dt = \frac{2}{3}t^3 - 5t^2 + 2t + x_0,$$

with initial condition of $x_0 = 0$ m,

$$x = \frac{2}{3}t^3 - 5t^2 + 2t.$$

The displacement as a function of time is $\frac{2}{3}t^3 - 5t^2 + 2t$.

2.4. How much mass can a 1 N force lift vertically?

Find: Mass m lifted by a force. Given force F = 1 N.

Properties: Gravitational acceleration g = 9.81 m/s^2.

Using the definition of weight and setting $F_w = F$,

$$m = \frac{F_w}{g} = \frac{1\text{ N}}{9.81\text{ m/s}^2} = 0.10194\text{ kg.}$$

The force can lift a 0.102 kg mass vertically.

2.5. What is the weight of an object with a mass of 150 kg on a planet where g = 4.1 m/s^2?

Find: Weight F_w of object.

Known: Mass of object m = 150 kg, gravitational acceleration g = 4.1 m/s^2.

Using the definition of weight,

$$F_w = mg = 150\text{ kg} \times 4.1\text{ m/s}^2 = 615\text{ N.}$$

The weight of the object is 615 N.

2.6. Humans subjected to accelerations greater than 5g may lose consciousness. What is the force acting on a 60 kg person at this acceleration?

Find: Force F acting on person.

Known: Acceleration a = 5g, mass of person m = 60 kg.

Properties: Gravitational acceleration g = 9.81 m/s^2.

Using the general equation of force,

$$F = ma = 60\text{ kg} \times 5 \times 9.81\text{ m/s}^2 = 2943\text{ N.}$$

This human may lose consciousness under a force greater than 2943 N.

2.7. A cylindrical water tank, 3 m high and 3 m in diameter is filled with water. If the density of water is 1000 kg/m^3, what is the mass of the contained water? If the acceleration due to gravity is 9.81 m/s^2, what is the weight of the water?

Find: Mass m and weight F_w of water in the tank.

Known: Tank height H = 3 m, tank diameter D = 3 m, density of water ρ = 1000 kg/m^3, gravitational acceleration g = 9.81 m/s^2.

The volume of the tank is

$$V = \frac{\pi D^2 H}{4} = \frac{\pi (3\text{ m})^2 \times 3\text{ m}}{4} = 21.2058\text{ m}^3,$$

And when filled with water contains a total mass of water of

$$m = \rho V = 1000\ \frac{\text{kg}}{\text{m}^3} \times 21.2058\text{ m}^3 = 21205.8\text{ kg}.$$

Then the weight of the water in the tank is

$$F_w = mg = 21205.8\text{ kg} \times 9.81\text{ m/s}^2 = 208{,}029\text{ N}.$$

The mass of water in the tank is 21200 kg, which has a weight of 208.0 kN.

2.8. A 5 kg box sliding across the floor with an initial velocity of 8 m/s is decelerated by friction to 3 m/s over 5 s. What is the force of friction acting on it?

Find: Frictional force F_f acting on the box.

Known: Mass of box m = 5 kg, initial velocity $\mathbf{V}_1$ = 8 m/s, final velocity $\mathbf{V}_2$ = 3 m/s, elapsed time Δt = 5 s.

Acceleration can be found using the definition in terms of velocity,

$$a = \frac{\Delta \mathbf{V}}{\Delta t} = \frac{3\text{ m/s} - 8\text{ m/s}}{5\text{ s}} = -1\text{ m/s}^2.$$

Then frictional force can be found using a force balance, since it is the only force acting on the box in the direction it is moving, and using the found acceleration:

$$F_f = ma = 5\text{ kg} \times (-1\text{ m/s}^2) = -5\text{ N}.$$

The frictional force opposing the motion of the box is 5 N .

2.9. A 500 N force accelerates a 50 kg mass moving at 10 m/s for 3 s. What is its final velocity?

Find: Final velocity $\mathbf{V}_2$.

Known: Force F = 500 N, mass m = 50 kg, initial velocity $\mathbf{V}_1$ = 10 m/s, force applied over elapsed time Δt = 3 s.

The acceleration can be found from a force balance, since there is only one force acting on the mass in the direction of motion:

$$a = \frac{F}{m} = \frac{500 \text{ N}}{50 \text{ kg}} = 10 \text{ m/s}^2.$$

Then the final velocity can be found by integrating the acceleration of the mass,

$$\mathbf{V}_2 = \int a dt = a\Delta t + \mathbf{V}_1 = 10 \text{ m/s}^2 \times 3 \text{ s} + 10 \text{ m/s} = 40 \text{ m/s}.$$

The final velocity of the mass after application of the force is 40 m/s.

2.10. A spaceship weighs 10,000 N on Earth. How much will it weigh on the moon where g = 1.64 m/s^2?

Find: Weight of spaceship on the moon $F_{w,m}$.

Known: Weight of spaceship on Earth $F_{w,e}$ = 10,000 N, gravitational acceleration on moon g_m = 1.64 m/s^2.

Properties: Gravitational acceleration on Earth g_e = 9.81 m/s^2.

The mass of the spaceship remains constant and can be found to be

$$m = \frac{F_{w,e}}{g_e} = \frac{10{,}000 \text{ N}}{9.81 \text{ m/s}^2} = 1019.368 \text{ kg}.$$

Then the weight of the spaceship on the moon can be found,

$$F_{w,m} = mg_m = 1019.368 \times 1.64 \text{ m/s}^2 = 1671.764 \text{ N}.$$

The spaceship has a weight of 1671.8 N on the moon.

2.11. A linear spring is one whose extension is proportional to the force applied. When a mass is suspended from a linear spring on Earth its extension is 6.3 mm. When the same mass is suspended from the spring on Mars, its extension is 2.5 mm. What is the acceleration due to gravity on Mars?

Find: Gravitational acceleration on Mars g_m.

Known: Linear spring extension on Earth x_e = 6.3 mm, spring extension on Mars x_m = 2.5 mm.

Properties: Gravitational acceleration on Earth g_e = 9.81 m/s^2.

From a force balance on a fully extended spring,

$$F = kx = mg,$$

where spring constant k and mass m remain constant on both planets. The extension on Earth is

$$x_e = \frac{m}{k} g_e,$$

and on Mars

$$x_m = \frac{m}{k} g_m,$$

so that:

$$\frac{x_m}{x_e} = \frac{g_m}{g_e},$$

$$g_m = \frac{x_m}{x_e} g_e = \frac{2.5 \text{ mm}}{6.3 \text{ mm}} \times 9.81 \text{ m/s}^2 = 3.8929 \text{ m/s}^2.$$

The gravitational acceleration on Mars is 3.89 m/s^2.

2.12. Acceleration due to gravity at Earth's surface is g = 9.80665 m/s^2 and decreases by approximately 3.3×10^{-6} m/s^2 for each metre of height above the ground. What is the potential energy of a 100 kg mass raised to an altitude of 1000 m: *a*) assuming constant *g*, *b*) accounting for the decrease in *g* with height?

Find: Potential energy *PE* of mass raised to an altitude with *a*) constant gravitational acceleration *g*, *b*) variable gravitational acceleration *g*.

Known: Gravitational acceleration g = 9.80665 m/s^2 – 3.3×10^{-6}z m/s^2, mass m = 100 kg, altitude z = 1000 m.

The mass starts at the ground, a state of zero gravitational potential energy, so $PE = \Delta PE$.

a) Assuming constant *g*:

Gravitational potential energy changes solely based on a change in altitude,

$$\Delta PE = mgh = 100 \text{ kg} \times 9.80665 \text{ m/s}^2 \times 1000 \text{ m} = 980665 \text{ J}.$$

b) Assuming variable *g* with altitude:

Intuition tells us that since *g* is decreasing with height, the final ΔPE value will be lower than in part a. From the definition of potential energy,

$$d(PE) = d(mgz),$$

$$\Delta PE = m\int_0^h g dz = m\int_0^h (9.80665 - 3.3\times10^{-6} z)dz,$$

$$\Delta PE = m\left[9.80665h - 1.65\times10^{-6} h^2\right] = 100 \times \left[9.80665(1000) - 1.65\times10^{-6}(1000)^2\right],$$

$$\Delta PE = 980500 \text{ J}.$$

The final potential energy of the mass is *a*) 980665 J assuming constant *g*, and *b*) 980500 J assuming variable *g*.

2.13. Acceleration due to gravity varies as $g = 9.80665\ \text{m/s}^2 - 3.3\times10^{-6}z\ \text{m/s}^2$, where z is the altitude in metres. What is the reduction in weight of a space shuttle as it rises from the Earth's surface to orbit at an altitude of 400 km?

Find: Weight reduction ΔF_w of the shuttle.

Known: Gravitational acceleration $g = 9.80665\ \text{m/s}^2 - 3.3\times10^{-6}z\ \text{m/s}^2$, initial height $z_0 = 0$ m, final height $z_1 = 400$ km.

On Earth's surface $g_0 = 9.80665\ \text{m/s}^2$. In orbit, at a height of $z_1 = 400$ km,

$$g_1 = 9.80665 - 3.3\times10^{-6}\times 400\times10^{3} = 8.48665\ \text{m/s}^2.$$

Then using the definition of weight,

$$F_w = mg,$$

$$\frac{\Delta F_w}{F_w} = \frac{g_1 - g_0}{g} = \frac{9.80665 - 8.48665}{9.80665} = 0.13460 = 13.5\%.$$

The weight reduction of the shuttle due to variable gravitational acceleration is 13.5%.

2.14. When a 6 kg mass is suspended from a spring whose extension (x) is proportional to the force applied (F), so that $F = Cx$, it stretches the spring by 30 mm. What is the proportionality constant C of the spring, in units of newtons per millimetre?

Find: Spring constant C (N/mm).

Known: Suspended mass $m = 6$ kg, extension $x = 30$ mm.

Using the given spring equation with an energy balance for an extended spring,

$$C = \frac{F}{x} = \frac{mg}{x} = \frac{6\ \text{kg}\times 9.81\ \text{m/s}^2}{30\ \text{mm}} = 1.9620\ \text{N/mm}.$$

The proportionality constant for this spring is 1.96 N/mm.

2.15. A car travelling at 40 km/h collides with a wall and comes to rest in 0.3 s. What is the force exerted by the seatbelt on the car driver, who has a mass of 75 kg?

Find: Force F exerted on car driver during collision.

Known: Car initial velocity $\mathbf{V}_1$ = 40 km/h, final velocity $\mathbf{V}_2$ = 0 km/h, elapsed time of deceleration Δt = 0.3s, mass of driver m = 75 kg.

The initial velocity of the car, converted into m/s, is

$$\mathbf{V}_1 = 40 \text{ km/h} = \frac{40 \text{ km/h} \times 1000 \text{ m/km}}{3600 \text{ s/h}} = 11.11111 \text{ m/s}.$$

The deceleration of the car and the driver was then

$$a = \frac{\Delta \mathbf{V}}{\Delta t} = \frac{11.11111 \text{ m/s} - 0}{0.3 \text{ s}} = 37.03703 \text{ m/s}^2,$$

and the force experienced by the car driver was

$$F = ma = 75 \text{ kg} \times 37 \text{ m/s}^2 = 2777.777 \text{ N}.$$

The force exerted on the car driver by the seatbelt was 2777.8 N.

2.16. An 1100 kg car undergoes constant acceleration from rest to a speed of 100 km/h in 8 s. What is its acceleration and how much force was applied to it?

Find: Acceleration a of car, force F applied to car.

Known: Mass of car m = 1100 kg, initial velocity $\mathbf{V}_1$ = 0 km/h, final velocity $\mathbf{V}_2$ = 100 km/h, elapsed time Δt = 8 s.

The final velocity of the car, in m/s, is

$$\mathbf{V} = 100 \text{ km/h} \times 1000 \text{ m/km} \times \frac{1}{3600 \text{ s/h}} = 27.77778 \text{ m/s}.$$

From the definition of acceleration in terms of velocity,

$$a = \frac{d\mathbf{V}}{dt} = \frac{\Delta \mathbf{V}}{\Delta t} = \frac{27.77778 \text{ m/s} - 0}{8 \text{ s}} = 3.472223 \text{ m/s}^2,$$

and the force required to obtain this acceleration is

$$F = ma = 1100 \text{ kg} \times 3.472223 \text{ m/s}^2 = 3819.445 \text{ N}.$$

The acceleration of the car is 3.47 m/s and the force applied to the car is 3819.4 N.