

CHAPTER II

PROBLEM #1

$$\nabla \times \bar{\mathcal{H}} = \frac{\partial \bar{\mathcal{D}}}{\partial t} \quad (2-15)$$

$$\nabla \times \nabla \times \bar{\mathcal{H}} = \nabla \times \frac{\partial \bar{\mathcal{D}}}{\partial t}$$

$$\nabla(\nabla \cdot \bar{\mathcal{H}}) - \nabla^2 \bar{\mathcal{H}} = \nabla \times \frac{\partial \bar{\mathcal{D}}}{\partial t}$$

$$\nabla(\nabla \cdot \bar{\mathcal{H}}) - \nabla^2 \bar{\mathcal{H}} = \nabla \times \frac{\partial}{\partial t} \epsilon \bar{\mathcal{E}}$$

$$= \frac{\partial}{\partial t} \epsilon \nabla \times \bar{\mathcal{E}}$$

$$= \frac{-\partial^2}{\partial t^2} \epsilon \mu_0 \bar{\mathcal{H}} \quad (\text{using Eq. 2-14})$$

$$\text{now } \nabla \cdot \mu_0 \bar{\mathcal{H}} = \mu_0 \nabla \cdot \bar{\mathcal{H}} = 0 \quad (\text{from 2-17})$$

$$\text{so } -\nabla^2 \bar{\mathcal{H}} = \frac{-\partial^2}{\partial t^2} \epsilon \mu_0 \bar{\mathcal{H}}$$

$$\text{or } \nabla^2 \bar{\mathcal{H}} - \epsilon \mu_0 \frac{\partial^2}{\partial t^2} \bar{\mathcal{H}} = 0 \quad (2-27)$$

PROBLEM #2

$$\nabla \times \bar{\mathcal{E}} = -j\omega\mu\bar{\mathcal{H}}$$

$$\nabla \times \nabla \times \bar{\mathcal{E}} = \nabla \times -j\omega\mu\bar{\mathcal{H}}$$

$$\nabla \times \nabla \times \bar{\mathcal{E}} = \omega^2 \mu \epsilon \bar{\mathcal{E}}$$

$$\nabla(\nabla \cdot \bar{\mathcal{E}}) - \nabla^2 \bar{\mathcal{E}} = \omega^2 \mu \epsilon \bar{\mathcal{E}}$$

$$-\nabla^2 \bar{\mathcal{E}} = \omega^2 \mu \epsilon \bar{\mathcal{E}}$$

$$\nabla^2 \bar{\mathcal{E}} + \omega^2 \mu \epsilon \bar{\mathcal{E}} = 0$$

$$\nabla^2 \bar{\mathcal{E}} + k^2 \bar{\mathcal{E}} = 0$$

PROBLEM #3

$$v = \frac{c}{n} = \frac{3 \times 10^8 \text{ m/sec}}{1.530} = 1.96 \times 10^8 \text{ m/sec}$$

$$\lambda = \frac{\lambda_0}{n} = 0.65 \text{ } \mu\text{m}$$

$$k = \frac{2\pi}{\lambda_0} n = \frac{2\pi}{1 \text{ } \mu\text{m}} (1.530) = 9.61 \times 10^6 / \text{m}$$

$$\eta = \sqrt{\frac{\mu_0}{\epsilon}} = \frac{1}{n} \sqrt{\frac{\mu_0}{\epsilon_0}} = \frac{1}{1.530} (377)$$

$$= 246 \Omega$$

Free space

$$v = c = 3 \times 10^8 \text{ m/sec}$$

$$\lambda = 1 \text{ } \mu\text{m}$$

$$k = \frac{2\pi}{\lambda_0} = 6.28 \times 10^6 / \text{m}$$

PROBLEM #4

I.

$$E_T = \tau_{\perp} E_i$$

$$\tau_{\perp} = \frac{2n_2 \cos \theta_1}{n_2 \cos \theta_1 + n_1 \cos \theta_2} \quad (2-128)$$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$\sin \theta_2 = \frac{1.5 \sin 30^\circ}{1.4}$$

$$\theta_2 = 32.39^\circ$$

$$\tau_{\perp} = \frac{2 \frac{1}{1.4} \cos 30^\circ}{\frac{1}{1.4} \cos 30^\circ + \frac{1}{1.5} \cos 32.39^\circ}$$

$$= 1.05$$

$$E_T = 1.05 [5e^{-j\beta_2 \hat{n}_2 \cdot \vec{r}}] \hat{a}_x$$

a) $|E_T| = |5.25|$ independent of z position

b) $\vec{S}_2 = \frac{1}{2n_2} |E_T|^2 (\hat{n}_2)$

$$= \frac{1}{2 \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_r}}} |5.25|^2 \left((\sin 32.39^\circ (-\hat{a}_y) + \cos 32.39^\circ \hat{a}_z) \right)$$

$$= 5.12 \times 10^{-2} (-0.54 \hat{a}_y + 0.84 \hat{a}_z)$$

$$= -2.74 \times 10^{-2} \hat{a}_y + 4.32 \times 10^{-2} \hat{a}_z$$

The z component is 4.32×10^{-2} watts.

PROBLEM #4 (Cont'd)

II.

$$\theta_c = \sin^{-1} n_2/n_1$$

$$= 68.96^\circ$$

PROBLEM #5

θ_1 is greater than the critical angle. Total internal reflection occurs and the wave in medium 2 is an evanescent wave.

$$E_T = \bar{E}_2 e^{-jk_2 \hat{n}_2 \cdot \vec{r}}$$

For an evanescent wave this becomes

$$E_T = \bar{E}_2 e^{-\alpha z} e^{j\beta y} \quad (2-151)$$

$$\alpha = \omega \sqrt{\mu_0 \epsilon_2} \sqrt{\frac{n_1^2}{n_2^2} \sin^2 \theta_1 - 1}$$

$$= 2.35 \times 10^6 / \text{m}$$

$$|E_T| = |E_2| e^{-\alpha z}$$

at $z = 10 \mu\text{m}$

$$|E_T| = |E_2| e^{-23.5}$$

$$= 6.48 \times 10^{-11} |E_2| \quad \text{V/m}$$

The power transmitted in the z direction is reactive. The z component of the Poynting vector is zero ($\hat{a}_z \cdot \vec{S}_T$) is imaginary.