

## CHAPTER

### 1

1.1-1.  $A = \{0 < \text{integers} < 4\}$ ,  $B = \{6 < \text{even integers} < 16\}$ ,  
 $C = \{0 < \text{odd integers}\}$ . Other definitions also are possible.

1.1-2. Class =  $\{A, B, C\}$  or class =  $\{\{1, 2, 3\}, \{8, 10, 12, 14\}, \{1, 3, 5, 7, \dots\}\}$ .

1.1-3. A, B, D, E, and F are countable and finite.  
C is countable and infinite. G, H, and I are uncountable and infinite.

1.1-4.  $A = B$ ,  $A \subset C$ ,  $A \subset G$ ,  $A \subset I$ .

$B = A$ ,  $B \subset C$ ,  $B \subset G$ ,  $B \subset I$ .

C is not equal to, or a subset of, any of the other sets. The same applies to D.

$E \subset D$ .

$F \subset D$ ,  $F \subset E$  (Note that F may be a null set.)

G is not equal to, or a subset of, any of the other sets.

$H \subset G$ , if x is in meters and negative length is allowed.

I is not equal to, or a subset of, any of the other sets.

1.1-5.  $\{\cdot\}$  (null set);  $\{a\}$ ,  $\{b\}$ ,  $\{c\}$ ,  $\{d\}$ ;  $\{a, b\}$ ,  $\{a, c\}$ ,  $\{a, d\}$ ,  $\{b, c\}$ ,  $\{b, d\}$ ,  $\{c, d\}$ ;  $\{a, b, c\}$ ,  $\{a, b, d\}$ ,  $\{a, c, d\}$ ,  $\{b, c, d\}$ ;  $\{a, b, c, d\}$ .

1.1-6. (a)  $S = \{-40^\circ\text{F} \leq t \leq 130^\circ\text{F}\}$ ,  $t$  is temperature.  
 (b)  $\{t \leq 32^\circ\text{F}\}$ . (c)  $\{32^\circ\text{F} < t \leq 100^\circ\text{F}\}$ .

\* 1.1-7. The null set is one subset. There are  $N$  subsets with one element each. Taking elements by pairs corresponds to  $\binom{N}{2} = \frac{N!}{2!(N-2)!}$  subsets, which is the number of combinations possible of  $N$  things taken 2 at a time. For subsets with three elements the number is  $\binom{N}{3} = \frac{N!}{3!(N-3)!}$ . Continuing the logic gives

$$\text{Subsets} = \sum_{i=0}^N \binom{N}{i} = \sum_{i=0}^N \frac{N!}{i!(N-i)!} = 2^N$$

where a series has been used from Jolley (1961, p. 36).

1.1-8. (a)  $S = \{-10 \leq A \leq 10\}$ . (b)  $V = \{0 \leq A \leq 10\}$ .  
 (c)  $S = \{-13 \leq A \leq 7\}$ ,  $V = \{0 \leq A \leq 7\}$ .

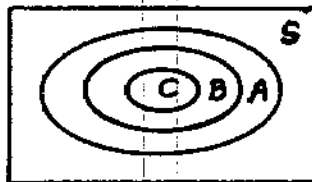
1.1-9.  $A = \{-7, -6, -5, \dots, -1, 0, 1, \dots, 5, 6, 7\}$ ,  $B = \{-1, 1, 3, 5\}$ ,  
 $A \subset B$  is not true,  $B \subset A$  is true.

1.1-10.  $A_1 = \{a_1\}$ ,  $A_2 = \{a_2\}$ ,  $A_3 = \{a_3\}$ ,  $A_4 = \{a_1, a_2\}$ ,  
 $A_5 = \{a_1, a_3\}$ ,  $A_6 = \{a_2, a_3\}$ ,  $A_7 = \{a_1, a_2, a_3\} = A$ ,  
 $A_8 = \phi$ .

- 1.1-11. (a)  $\{2, 3, 4, 5, 6, 7, 8\}$ ,  $\{1 < I = \text{integers} < 9\}$ .  
 (b)  $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ ,  $\{1 \leq I = \text{integers} \leq 9\}$ .  
 (c)  $R_{\text{equivalent}} = R/n$ ,  $n=1, 2, \dots, 5$ , where  $R = 10 \sqrt{2}$ , so  $\{10, 5, 10/3, 2.5, 2\}$ ,  $\{R = 10/n, n=1, 2, \dots, 5\}$ . (d)  $R_{\text{equivalent}} = nR$ ,  $n=1, 2, \dots, 6$ , where  $R = 2.2 \sqrt{2}$ , so  $\{2.2, 4.4, 6.6, 8.8, 11.0, 13.2\}$ ,  $\{R = 2.2n, n=1, 2, \dots, 6\}$ .

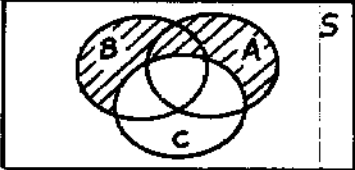
1.1-12. (a) 60, (b) 100, (c) 35.

1.2-1. A Venn diagram proves  $C \subset A$  if  $C \subset B$  and  $B \subset A$ .

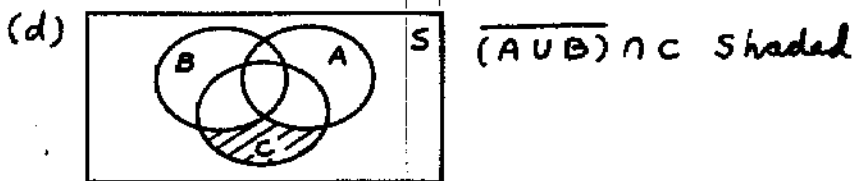
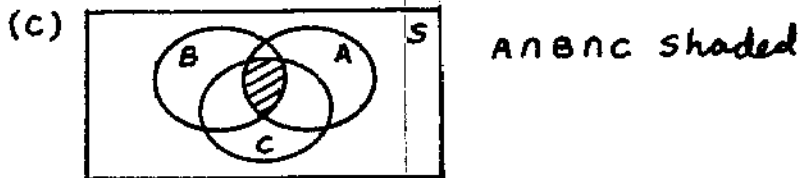


1.2-2. (a)  $A - B = \{-6, -4, 1.6, 8\}$ . (b)  $B - A = \{1, 2, 4\}$ .  
 (c)  $A \cup B = \{-6, -4, -0.5, 0, 1, 1.6, 2, 4, 8\}$ .  
 (d)  $A \cap B = \{-0.5, 0\}$ .

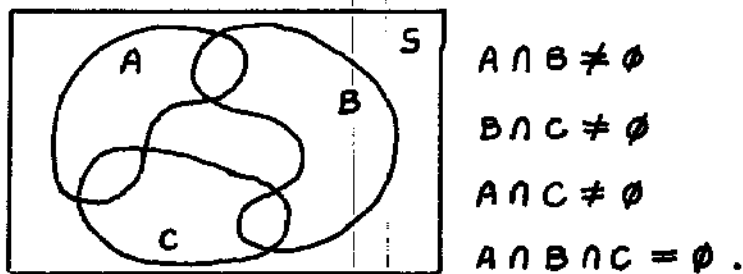
1.2-3. (a)  $\bar{A} = S - A = \{6, 8, 12\}$ . (b)  $A - B = \{2\}$ ;  
 $B - A = \{6, 8\}$ . (c)  $A \cup B = \{2, 4, 6, 8, 10\}$ .  
 (d)  $A \cap B = \{4, 10\}$ . (e)  $\bar{A} \cap B = \{6, 8\}$ .

1.2-4. (a)  (A  $\cup$  B) - C shaded

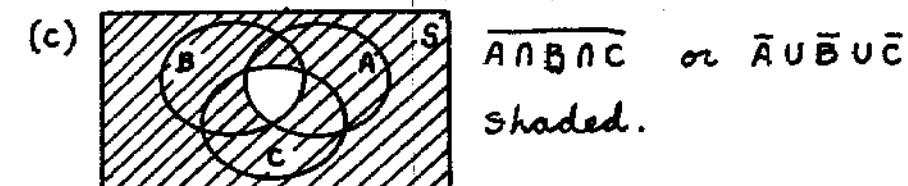
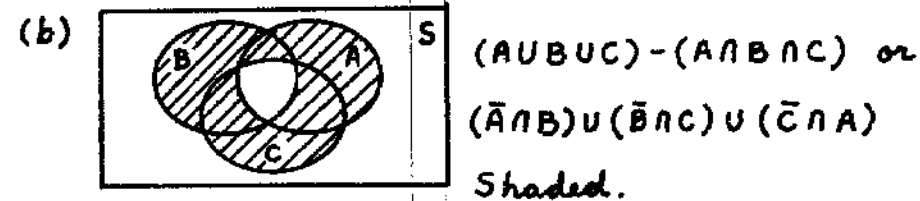
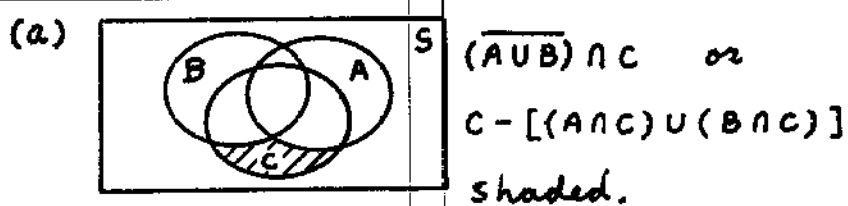
1.2-4. (Continued)



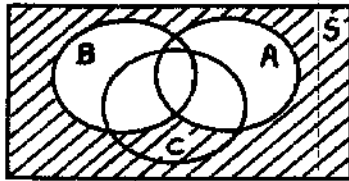
1.2-5.



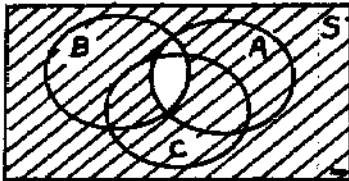
1.2-6.



1.2-7.



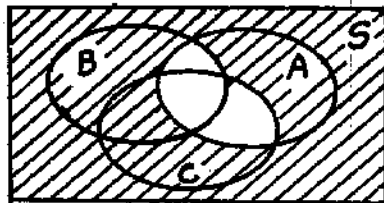
$\overline{A \cup B}$  or  
 $\bar{A} \cap \bar{B}$  shaded.



$\overline{A \cap B}$  or  
 $\bar{A} \cup \bar{B}$  shaded.

1.2-8. (a)  $A \cup B = \{-10 \leq x < -4\}$ . (b)  $A \cap B = \{-7 < x \leq -5\}$ . (c) The set  $C = \{-9 \leq x \leq N\}$  satisfies making  $A \cap C$  as large as possible for any  $-5 \leq N \leq -4$ . The set  $B \cap C$  is largest if  $C = \{-9 \leq x \leq -4\}$ . The set satisfying both requirements is therefore  $C = \{-9 \leq x \leq -4\}$ . The set  $C = \{-9 \leq x < -4\}$  is also a valid solution. (d)  $A \cap B \cap C = \{-7 < x \leq -5\}$ .

1.2-9. (a) To prove  $\overline{A \cap (B \cup C)} = (\bar{A} \cup \bar{B}) \cap (\bar{A} \cup \bar{C})$  work with the left side. Use (1.2-13) to get  $\overline{A \cap (B \cup C)} = \bar{A} \cup \overline{(B \cup C)}$ . Next, use (1.2-12) to get  $\overline{A \cap (B \cup C)} = \bar{A} \cup (\bar{B} \cap \bar{C})$ . Finally, (1.2-9) is used to obtain the desired equation.

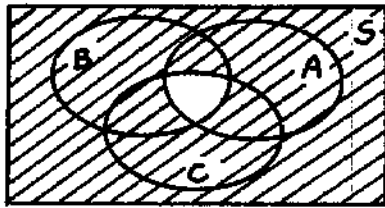


$\overline{A \cap (B \cup C)}$  and  
 $(\bar{A} \cup \bar{B}) \cap (\bar{A} \cup \bar{C})$  shaded.

(b) By using (1.2-13) on the left side of  $\overline{A \cap B \cap C} =$

1.2-9. (Continued)

$\bar{A} \cup \bar{B} \cup \bar{C}$  we have  $\overline{A \cap B \cap C} = \bar{A} \cup (\bar{B} \cap \bar{C}) = \bar{A} \cup (\bar{B} \cup \bar{C}) = \bar{A} \cup \bar{B} \cup \bar{C}$ .

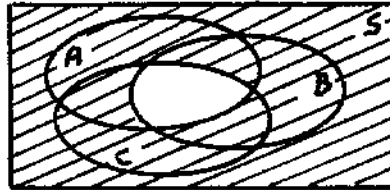


$\overline{A \cap B \cap C}$  and  $\bar{A} \cup \bar{B} \cup \bar{C}$  shaded.

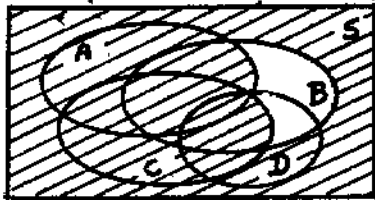
1.2-10.



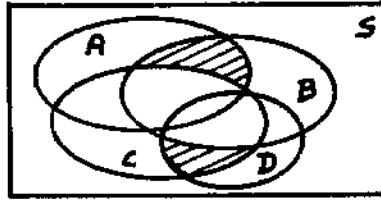
(a)  $(A \cup B) \cap \bar{C}$



(b)  $\overline{(A \cap B)} \cup \bar{C}$

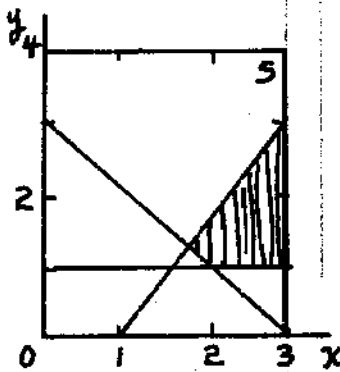


(c)  $(A \cup \bar{B}) \cup (C \cap D)$

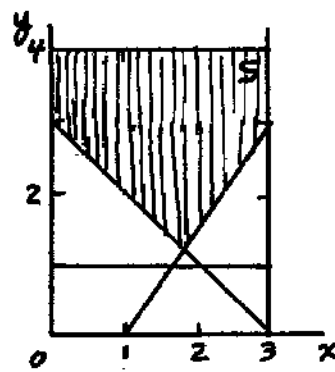


(d)  $(A \cap B \cap \bar{C}) \cup (\bar{B} \cap C \cap D)$

1.2-11.



(a)  $A \cap B \cap C$



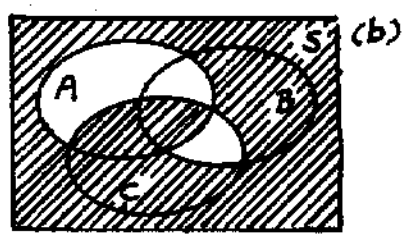
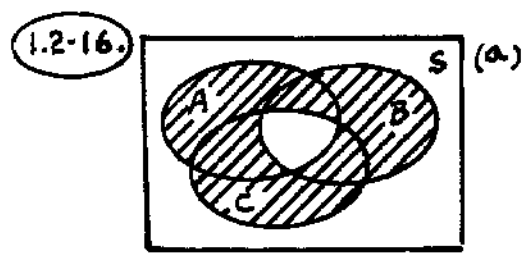
(b)  $C \cap B \cap \bar{A}$

1.2-12. (a)  $A = \{80 \text{ m} \leq d \leq 1050 \text{ m}\}$ ,  $B = \{950 \text{ m} \leq d \leq 1750 \text{ m}\}$ ,  
 $C = \{1750 \text{ m} \leq d \leq 2000 \text{ m}\}$ . (b)  $A \cap B = \{950 \text{ m} \leq d \leq 1050 \text{ m}\}$   
 is the set of distances where both propeller and jet  
 aircraft take off. (c)  $\overline{A \cup B} = \{0 \leq d \leq 80 \text{ m and } 1750 \text{ m}$   
 $< d \leq 2000 \text{ m}\}$  is the portion of the runway used by  
 both types of aircraft and the portion used by neither  
 (safety margin). (d)  $\overline{A \cup B \cup C} = \{0 \leq d < 80 \text{ m}\}$  is portion  
 of runway used by all aircraft in takeoff.  $A \cup B =$   
 $\{80 \text{ m} \leq d \leq 1750 \text{ m}\}$  is the runway distances used by  
 all aircraft in taking off.

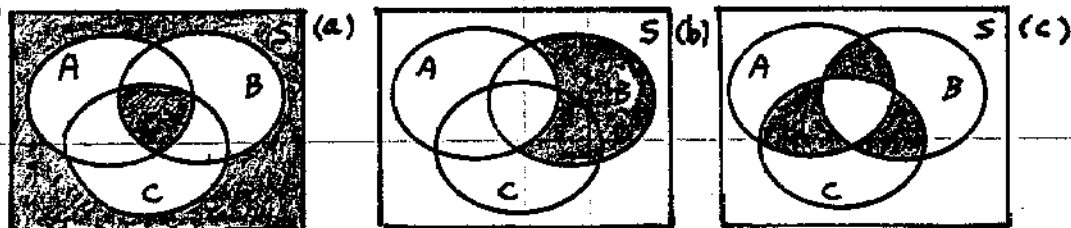
1.2-13.  $\overline{A_1 \cap A_2} = \overline{A_1} \cup \overline{A_2} \triangleq C$  so  $A_1 \cap A_2 = \overline{C}$   
 $\overline{A_1 \cap A_2 \cap A_3} = \overline{\overline{C} \cap A_3} = C \cup \overline{A_3} = \overline{A_1} \cup \overline{A_2} \cup \overline{A_3}$   
 Continuation of the iteration proves the desired result.

1.2-14. Apply (1.2-12) and define  $C$  as  
 $\overline{A_1 \cup A_2} = \overline{A_1} \cap \overline{A_2} \triangleq C$  so  $\overline{C} = A_1 \cup A_2$   
 $\overline{A_1 \cup A_2 \cup A_3} = \overline{\overline{C} \cup A_3} = C \cap \overline{A_3} = \overline{A_1} \cap \overline{A_2} \cap \overline{A_3}$   
 Continued iteration proves the desired result.

1.2-15. (a) False, (b) true, (c) false, (d) false,  
 (e) true, (f) true, and (g) false.



1.2-17.



1.3-1. The events are  $A = \{1, 3, 5\}$ ,  $B = \{4, 5, 6\}$ ,  $A \cup B = \{1, 3, 4, 5, 6\}$ ,  $A \cap B = \{5\}$ . By assuming a fair die the probability of each of the six mutually exclusive outcomes is  $1/6$ . Thus, from axiom 3:  $P(A) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 1/2$ ,  $P(B) = 1/2$ ,  $P(A \cup B) = 5/6$  and  $P(A \cap B) = 1/6$ .

1.3-2. There are 36 possible mutually exclusive possible outcomes, each with probability  $1/36$ . Only six produce a seven:  $(1,6)$ ,  $(2,5)$ ,  $(3,4)$ ,  $(4,3)$ ,  $(5,2)$  and  $(6,1)$ . Two produce an eleven:  $(5,6)$  and  $(6,5)$ . Thus, eight outcomes satisfy the event "7 or 11." Therefore  $P(\text{sum is 7 or 11}) = 8/36 = 2/9$ .

1.3-3. (a)  $S = \{0 < \lambda \leq 100\}$ . (b)  $P\{20 < \lambda \leq 35\} = (35-20)/100 = 15/100$ . (c)  $P\{\lambda = 58\} = 0$  since the number 58 is only one of an infinite number of numbers in  $S$ .

1.3-4. (a)  $P(A \cup C) = P\{a_1, a_5, a_6, a_9\} = 4/10$ . (b)  $P(B \cup \bar{C}) = P\{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}\} = P(S) = 1$ . (c)  $P[A \cap (B \cup C)] = P\{a_1, a_9\} = 2/10$ .



1.3-4. (Continued)

(d)  $P(\overline{A \cup B}) = P\{a_3, a_4, a_7, a_8, a_{10}\} = 5/10 = 1/2.$

(e)  $P[A \cup (B \cap C)] = P\{a_6, a_9\} = 2/10 = 1/5.$

1.3-5. A and  $\bar{A}$  are mutually exclusive and span the entire sample space. Therefore, from axiom 3:  $P(A \cup \bar{A}) = P(S) = 1 = P(A) + P(\bar{A})$  or  $P(\bar{A}) = 1 - P(A).$

1.3-6. (a)  $P(A) = P\{6\} = 1/6$ ;  $P(B) = P\{2, 5\} = 2/6 = 1/3.$

(b)  $P(A) + P(B) + P(C) = 1$  is guaranteed if C is exclusive of A and B and comprises the balance of the sample space. Thus,  $C = \overline{A \cup B} = \{1, 3, 4\}.$

1.3-7.  $P(\text{black}) = 75/500$ ,  $P(\text{green}) = 150/500$ ,  $P(\text{red}) = 175/500$ ,  $P(\text{white}) = 70/500$  and  $P(\text{blue}) = 30/500.$

1.3-8. (a)  $P(\text{jack}) = 4 \text{ jacks} / 52 \text{ cards} = 4/52 = 1/13.$

(b)  $P(5 \text{ or smaller}) = [4 \text{ fives} + 4 \text{ fours} + 4 \text{ threes} + 4 \text{ twos}] / 52 \text{ cards} = 16/52 = 4/13.$  (c)  $P(\text{red ten}) = [1 \text{ ten of hearts} + 1 \text{ ten of diamonds}] / 52 \text{ cards} = 2/52.$

1.3-9. (a)  $P(A \text{ wins}) = P(2, 4) + P(1, 4) + P(4, 1) + P(4, 2) = 4/36,$  (b)  $P(B \text{ wins}) = P(4, 1) + P(4, 2) + P(4, 3) + P(4, 4) + P(4, 5) + P(4, 6) + P(1, 4) + P(2, 4) + P(3, 4) + P(5, 4) + P(6, 4) = 11/36.$  (c)  $P(A \text{ and } B \text{ win}) = P(A \text{ wins}) = 4/36$  because  $A \subset B.$

1.3-10. (a)  $P(\text{stay in after one toss}) = P(H, H, T) + P(H, T, H) + P(T, T, H) + P(T, H, T) = 4/8$ . (b)  $P(\text{out after first toss}) = P(H, T, T) + P(T, H, H) = 2/8$ . (c)  $P(\text{no "odd man"}) = P(H, H, H) + P(T, T, T) = 2/8$ .

1.3-11.

$\Omega \backslash W$	0.25	0.50	1.0
10	0.08	0.10	0.01
22	0.20	0.26	0.05
48	0.12	0.15	0.03
	0.40	0.51	0.09

$S = \{ \text{all resistors} \}$

All resistors span  $S$ .

From table: (a)  $P(48\Omega \text{ and } 0.25W) = 0.12$ , (b)  $P(48\Omega \text{ and } 0.5W) = 0.15$ , (c)  $P(48\Omega \text{ and } 1.0W) = 0.03$ .

1.3-12.

(a)  $P(\text{one is a two and other is 3 or more}) = P(2, 3) + P(2, 4) + P(2, 5) + P(2, 6) + P(3, 2) + P(4, 2) + P(5, 2) + P(6, 2) = 8(1/36) = 2/9$ . (b)  $P(10 \leq \text{sum and sum} \leq 4) = P(4, 6) + P(5, 5) + P(5, 6) + P(6, 4) + P(6, 5) + P(6, 6) + P(1, 1) + P(1, 2) + P(1, 3) + P(2, 1) + P(2, 2) + P(3, 1) = 12/36 = 1/3$ .

1.3-13.

Probabilities of sums showing up are:

	1	2	3	4	5	6
1			$2/49$			
2			$2/49$			
3			$2/49$			
4	$2/49$	$2/49$	$4/49$	$2/49$	$2/49$	$2/49$
5			$2/49$			
6			$2/49$			

all other probabilities not shown are  $1/49$

$$P\{7\} = P(6,1) + P(5,2) + P(4,3) + P(3,4) + P(2,5) + P(1,6) = 9/49 \approx 0.1837$$

For fair dice each outcome has probability =  $1/36$  so

$$P\{7\} = 6/36 = 1/6 \approx 0.1667. \text{ Improvement} = (9/49)/(6/36) \approx 1.102, (10.2\%).$$

1.4-1. (a)  $P(\text{second is queen} | \text{first is queen}) = 3/51.$

(b)  $P(\text{second is seven} | \text{first is queen}) = 4/51.$

(c)  $P(\text{queen} \cap \text{queen}) = P(\text{queen} | \text{queen}) P(\text{queen}) = \frac{3}{51} \cdot \frac{4}{52} = 1/221.$

1.4-2.  $P(\text{four sevens}) = P(\text{fourth is seven} | \text{first three sevens})$

$\cdot P(\text{first three sevens}) = \frac{1}{49} P(\text{first three sevens}) =$

$\frac{1}{49} P(\text{third is seven} | \text{first two sevens}) P(\text{first two sevens})$

$= \frac{1}{49} \cdot \frac{2}{50} \cdot P(\text{first two sevens}) = \frac{1}{49} \cdot \frac{2}{50} P(\text{second is seven} | \text{first is seven}) P(\text{first is seven}) = \frac{1}{49} \cdot \frac{2}{50} \cdot \frac{3}{51} \cdot \frac{4}{52} = \frac{1}{270725}$

$= 3.694 (10^{-6}).$

1.4-3.  $P(D) = 24/100, P(E) = 38/100,$

$P(D \cap E) = 14/100, P(D|E) = 14/38,$

$P(E|D) = 14/24.$

1.4-4. Define  $B_1$  = "draw a 5% resistor" and  $B_2$  = "draw a 10% resistor." (a) From (1.4-10):

$$P(D) = P(D|B_1)P(B_1) + P(D|B_2)P(B_2) \\ = \frac{10}{62} \left( \frac{62}{100} \right) + \frac{14}{38} \left( \frac{38}{100} \right) = 24/100.$$

$$(b) P(5\% | 22 \Omega) = P(B_1|D) = P(D|B_1)P(B_1)/P(D) \\ = \frac{10}{62} \left( \frac{62}{100} \right) / \frac{24}{100} = 10/24.$$

1.4-5. (a)  $P(0.01 \mu F | \text{box 2}) = 95/210.$

(b)  $P(0.01 \mu F | \text{box 3}) P(\text{box 3}) = P(\text{box 3} | 0.01 \mu F) P(0.01 \mu F)$

Thus,

$$P(\text{box 3} | 0.01 \mu F) = \frac{P(0.01 \mu F | \text{box 3}) P(\text{box 3})}{P(0.01 \mu F)}$$

From the total probability theorem:

$$P(0.01 \mu F) = P(0.01 \mu F | \text{box 1}) P(\text{box 1}) \\ + P(0.01 \mu F | \text{box 2}) P(\text{box 2}) + P(0.01 \mu F | \text{box 3}) P(\text{box 3}) \\ = \frac{20}{145} \left( \frac{1}{3} \right) + \frac{95}{210} \left( \frac{1}{3} \right) + \frac{25}{245} \left( \frac{1}{3} \right) = \frac{5903}{3(29)42(7)}.$$

Thus,

$$P(\text{box 3} | 0.01 \mu F) = \frac{\frac{25}{245} \left( \frac{1}{3} \right)}{\frac{5903}{3(29)42(7)}} = \frac{870}{5903} \approx 0.1474.$$

1.4-6.  $P(0.01 \mu F | \text{box 1}) = \frac{20}{145}$ ,  $P(0.01 \mu F | \text{box 2}) = \frac{95}{210}$ ,

$$P(0.01 \mu F | \text{box 3}) = \frac{25}{245}, P(0.1 \mu F | \text{box 1}) = \frac{55}{145}, P(0.1 \mu F | \text{box 2}) = \frac{35}{210},$$

$$P(0.1 \mu F | \text{box 3}) = \frac{75}{245}, P(1.0 \mu F | \text{box 1}) = \frac{70}{145}, P(1.0 \mu F | \text{box 2}) = \frac{80}{210},$$

$$P(1.0 \mu F | \text{box 3}) = \frac{145}{245}.$$

1.4-7. From equations in Example 1.4-2:  $P(A_1) = 0.95(0.6) + 0.05(0.4) = 0.59$ ,  $P(A_2) = 0.05(0.6) + 0.95(0.4) = 0.41$ ,  $P(B_1|A_1) = 0.95(0.6)/0.59 = 0.966$ ,  $P(B_2|A_2) = 0.95(0.4)/0.41 = 0.927$ ,  $P(B_1|A_2) = 0.05(0.6)/0.41 = 0.073$ ,  $P(B_2|A_1) = 0.05(0.4)/0.59 = 0.034$ .

1.4-8. From equations in Example 1.4-2:  $P(A_1) = 1.0(0.7) + 0.0(0.3) = 0.7$ ,  $P(A_2) = 0.0(0.7) + 1.0(0.3) = 0.3$ ,  $P(B_1|A_1) = 1.0(0.7)/0.7 = 1.0$ ,  $P(B_2|A_2) = 1.0(0.3)/0.3 = 1.0$ ,  $P(B_1|A_2) = 0$ ,  $P(B_2|A_1) = 0$ . The system is ideal (or noise-free) since the probabilities of an error in symbol reception are zero.

1.4-9.

Event	10W	25W	50W	Totals
D (Defective)	15	7	3	25
G (Good)	85	63	27	175
Totals	100	70	30	200

(a)  $P(D|10W) = 15/100$ . (b)  $P(D \cap 50W) = P(D|50W)$

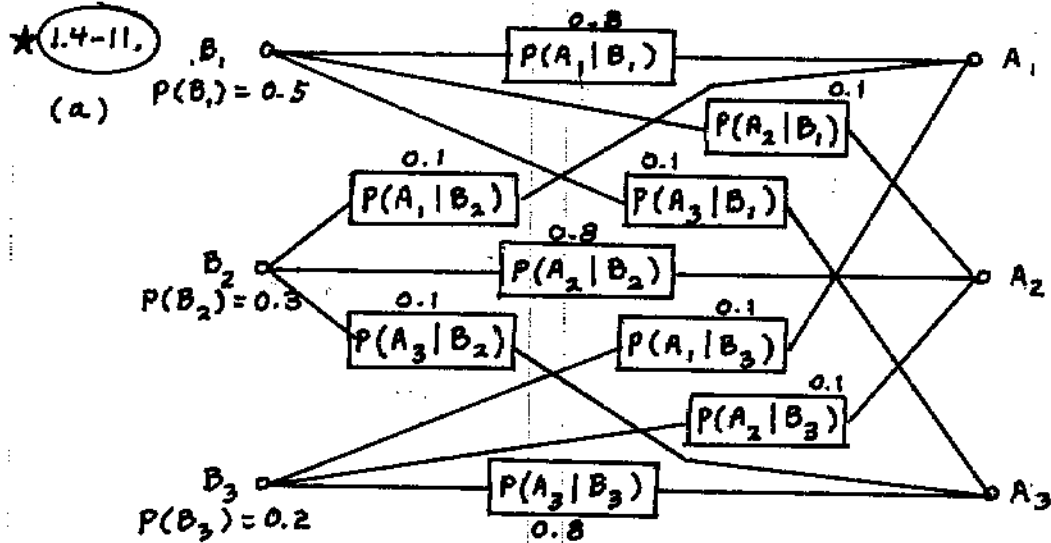
$\cdot P(50W) = \frac{3}{30} \left( \frac{1}{3} \right) = 1/30$ . (c)  $P(D) =$

$P(D|10W)P(10W) + P(D|25W)P(25W) + P(D|50W)P(50W)$   
 $= \frac{15}{100} \left( \frac{1}{3} \right) + \frac{7}{70} \left( \frac{1}{3} \right) + \frac{3}{30} \left( \frac{1}{3} \right) = 0.35/3 \approx 0.1167$ .

1.4-10. (a)  $P(\text{launch}) = P(A \text{ fails} \cap B \text{ fails})$   
 $= P(B \text{ fails} | A \text{ fails}) P(A \text{ fails}) = 0.06(0.01)$   
 $= 0.0006.$

(b)  $P(A \text{ fails} | B \text{ fails}) = P(A \text{ fails} \cap B \text{ fails}) / P(B \text{ fails}) =$   
 $6(10^{-4}) / 3(10^{-2}) = 0.02.$

(c)  $P(A \text{ fails}) P(B \text{ fails}) = 0.01(0.03) = 3(10^{-4}) \neq 6(10^{-6}) =$   
 $P(A \text{ fails} \cap B \text{ fails})$  so events "A fails" and  
 "B fails" are not independent.



(b)  $P(A_1) = P(A_1|B_1)P(B_1) + P(A_1|B_2)P(B_2) + P(A_1|B_3)P(B_3)$   
 $= 0.8(0.5) + 0.1(0.3) + 0.1(0.2) = 0.45$ ,  $P(A_2) =$   
 $0.1(0.5) + 0.8(0.3) + 0.1(0.2) = 0.31$ ,  $P(A_3) = 0.1(0.5)$   
 $+ 0.1(0.3) + 0.8(0.2) = 0.24.$  (c)

(c)  $P(B_1|A_1) = 0.8(0.5)/0.45 = 0.8889$ , Bayes' rule.

$P(B_1|A_2) = 0.1(0.5)/0.31 = 0.1613$

$P(B_1|A_3) = 0.1(0.5)/0.24 = 0.2083$

★ 1.4-11. (Continued)

$$P(B_2|A_1) = 0.1(0.3)/0.45 = 0.0667$$

$$P(B_2|A_2) = 0.8(0.3)/0.31 = 0.7742$$

$$P(B_2|A_3) = 0.1(0.3)/0.24 = 0.1250$$

$$P(B_3|A_1) = 0.1(0.2)/0.45 = 0.0444$$

$$P(B_3|A_2) = 0.1(0.2)/0.31 = 0.0645$$

$$P(B_3|A_3) = 0.8(0.2)/0.24 = 0.6667$$

(a) When  $P(B_i) = 1/3$ ,  $i = 1, 2, 3$ , then  $P(A_i) = 1/3 [P(A_i|B_1) + P(A_i|B_2) + P(A_i|B_3)] = \frac{1}{3} [0.8 + 0.1 + 0.1] = 1/3$ .

Similarly  $P(A_2) = P(A_3) = P(A_i) = 1/3$  and also

$P(B_i|A_k) = 0.1$ ,  $k \neq i$ , and  $P(B_i|A_i) = 0.8$ ,  $i = 1, 2, 3$ .

1.4-12.

	No. Pills	Line L	
		L <sub>1</sub>	L <sub>2</sub>
A <sub>1</sub>	102	0.02	0.03
A <sub>2</sub>	101	0.06	0.08
A <sub>3</sub>	100	0.88	0.83
A <sub>4</sub>	99	0.03	0.04
A <sub>5</sub>	98	0.01	0.02

$$P(L_1) = 0.45, \quad P(L_2) = 0.55$$

$$P(102|L_1) = 0.02, \quad P(102|L_2) = 0.03$$

$$P(101|L_1) = 0.06, \quad P(101|L_2) = 0.08$$

$$P(100|L_1) = 0.88, \quad P(100|L_2) = 0.83$$

$$P(99|L_1) = 0.03, \quad P(99|L_2) = 0.04$$

$$P(98|L_1) = 0.01, \quad P(98|L_2) = 0.02$$

(a)  $P(102) = P(102|L_1)P(L_1) + P(102|L_2)P(L_2) = 0.02(0.45) + 0.03(0.55) = 0.0255$ . Similarly,  $P(101) = 0.0710$ ,  $P(100) = 0.8525$ ,  $P(99) = 0.0355$ , and  $P(98) = 0.0155$ . (b)  $P(L_1|100) = \frac{P(100|L_1)P(L_1)}{P(100)} =$

(1.4-12.) (Continued)  $\frac{0.88(0.45)}{0.8525} \approx 0.4645$ . (c)  $P(\text{pills} < 100) =$

$P(98) + P(99) = 0.0355 + 0.0155 = 0.0510$ .

(1.4-13.)

Define:

$A = \{ \text{IC from source A} \}$

$B = \{ \text{IC from source B} \}$

$C = \{ \text{IC from source C} \}$

$D = \{ \text{defective IC} \}$

Then:

$P(D|A) = 0.001, P(A) = 1/3$

$P(D|B) = 0.003, P(B) = 1/3$

$P(D|C) = 0.002, P(C) = 1/3$

(a)  $P(D) = P(D|A)P(A) + P(D|B)P(B) + P(D|C)P(C) = (0.001/3) +$

$(0.003/3) + (0.002/3) = 0.006/3 = 0.002$ . (b)  $P(A|D) =$

$P(D|A)P(A)/P(D) = 0.001(1/3)/0.002 = 1/6$ . Similarly,

$P(B|D) = 1/2, P(C|D) = 1/3$ .

(1.4-14.)

(a)  $P(\text{acc}) = P(A|D_1)P(D_1) + P(A|D_2)P(D_2) + P(A|D_3)P(D_3) = \frac{4}{52}(\frac{1}{2})$

$+ (\frac{1}{3}) + 0(\frac{1}{6}) = \frac{19}{156} \approx 0.1218$ . (b)  $P(3) = P(3|D_1)P(D_1) + P(3|D_2)P(D_2)$

$+ P(3|D_3)P(D_3) = \frac{4}{52}(\frac{1}{2}) + 0(\frac{1}{3}) + \frac{4}{36}(\frac{1}{6}) = \frac{20}{351} \approx 0.0570$ . (c) Similarly,

$P(\text{red card}) = \frac{26}{52}(\frac{1}{2}) + \frac{8}{16}(\frac{1}{3}) + \frac{18}{36}(\frac{1}{6}) = \frac{1}{2} = 0.50$ .

(1.5-1.)  $P(A \cap B) = 28/100 = 0.28 \neq P(A)P(B) = (44/100)$

$\cdot (62/100) = 0.273$  so A and B are not independent.

$P(A \cap C) = 0.0$  while  $P(A)P(C) \neq 0$  so A and C are not independent.  $P(B \cap C) = 0.24 \neq P(B)P(C) =$

$(62/100)(32/100) = 0.198$  so B and C are not



1.5-1. (Continued)

independent. Finally,  $P(A \cap B \cap C) = 0.0 \neq P(A) \cdot P(B) \cdot P(C)$  so  $A, B,$  and  $C$  are not statistically independent, even by pairs.

1.5-2.  $P(A_1 \cap A_2) = P(A_1)P(A_2), \quad P(A_1 \cap A_3) = P(A_1)P(A_3),$   
 $P(A_1 \cap A_4) = P(A_1)P(A_4), \quad P(A_2 \cap A_3) = P(A_2)P(A_3),$   
 $P(A_2 \cap A_4) = P(A_2)P(A_4), \quad P(A_3 \cap A_4) = P(A_3)P(A_4),$   
 $P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2)P(A_3), \quad P(A_1 \cap A_2 \cap A_4) = P(A_1)P(A_2)P(A_4),$   
 $P(A_1 \cap A_3 \cap A_4) = P(A_1)P(A_3)P(A_4), \quad P(A_2 \cap A_3 \cap A_4) = P(A_2)P(A_3)P(A_4),$   
and  $P(A_1 \cap A_2 \cap A_3 \cap A_4) = P(A_1)P(A_2)P(A_3)P(A_4).$

★ 1.5-3. (a) First form the product  $P(A_1)P(\bar{A}_2) = P(A_1)[1 - P(A_2)]$   
 $= P(A_1) - P(A_1)P(A_2)$ . Since  $A_1$  and  $A_2$  are independent this becomes

$$P(A_1)P(\bar{A}_2) = P(A_1) - P(A_1 \cap A_2). \quad (1)$$

A Venn diagram shows that  $A_1 = (A_1 \cap \bar{A}_2) \cup (A_1 \cap A_2)$   
so  $P(A_1) = P[(A_1 \cap \bar{A}_2) \cup (A_1 \cap A_2)]$ . From (1.4-2):

$$P[(A_1 \cap \bar{A}_2) \cup (A_1 \cap A_2)] = P(A_1 \cap \bar{A}_2) + P(A_1 \cap A_2)$$

$- P[(A_1 \cap \bar{A}_2) \cap (A_1 \cap A_2)]$ . However, a Venn diagram will show that  $(A_1 \cap \bar{A}_2) \cap (A_1 \cap A_2) = \emptyset$  so these last two results give

$$P(A_1) = P(A_1 \cap \bar{A}_2) + P(A_1 \cap A_2). \quad (2)$$

By combining (1) and (2) we have  $P(A_1)P(\bar{A}_2) = P(A_1 \cap \bar{A}_2)$  so  $A_1$  and  $\bar{A}_2$  are statistically indep-

\*1.5-3. (Continued)

endent from (1.5-3).

(b) The procedure is the same as in (a):  $P(\bar{A}_1)P(A_2) = P(A_2)[1 - P(A_1)] = P(A_2) - P(A_1 \cap A_2)$ . But  $A_2 = (A_2 \cap \bar{A}_1) \cup (A_2 \cap A_1)$  and  $(A_2 \cap \bar{A}_1) \cap (A_2 \cap A_1) = \emptyset$  so  $P(A_2) = P[(A_2 \cap \bar{A}_1) \cup (A_2 \cap A_1)] = P(A_2 \cap \bar{A}_1) + P(A_2 \cap A_1) - P[(A_2 \cap \bar{A}_1) \cap (A_2 \cap A_1)] = P(A_2 \cap \bar{A}_1) + P(A_2 \cap A_1)$ . Thus, from the first and last equations:

$$P(\bar{A}_1)P(A_2) = P(\bar{A}_1 \cap A_2)$$

so  $\bar{A}_1$  and  $A_2$  are independent.

(c) Again repeat the procedures of (a) and (b):

$$P(\bar{A}_1)P(\bar{A}_2) = [1 - P(A_1)][1 - P(A_2)] = 1 - P(A_1) - P(A_2)$$

$$+ P(A_1)P(A_2) = 1 - P(A_1) - P(A_2) + P(A_1 \cap A_2)$$

$= 1 - [P(A_1) + P(A_2) - P(A_1 \cap A_2)] = 1 - P(A_1 \cup A_2)$  from (1.4-2). But since  $1 - P(A_1 \cup A_2) = P(\overline{A_1 \cup A_2})$ , then  $P(\bar{A}_1)P(\bar{A}_2) = P(\overline{A_1 \cup A_2})$ . However, a Venn diagram shows that  $\overline{A_1 \cup A_2} = \bar{A}_1 \cap \bar{A}_2$  so

$$P(\bar{A}_1)P(\bar{A}_2) = P(\bar{A}_1 \cap \bar{A}_2)$$

which, from (1.5-3) shows  $\bar{A}_1$  and  $\bar{A}_2$  to be independent.

1.5-4. Comb.  $N$  things 2 at time =  $\binom{N}{2}$  (pairs)

Comb.  $N$  things 3 at time =  $\binom{N}{3}$  (triples)

$\vdots$

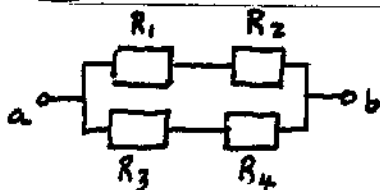
Comb.  $N$  things  $N$  at time =  $\binom{N}{N}$  ( $N$ -tuple)

1.5-4. (Continued)

$$\text{Total} = \text{sum} = \sum_{i=2}^N \binom{N}{i} = -\binom{N}{0} - \binom{N}{1} + \sum_{i=0}^N \binom{N}{i}$$

$$\text{Use (C-61). Total} = -1 - N + \sum_{i=0}^N \binom{N}{i} = 2^N - N - 1.$$

1.5-5.



$$R_i = \{ \text{relay } R_i \text{ fails, } i=1, 2, 3, 4 \} = \{ R_i \text{ opens} \}$$

$$P(R_i) = \begin{cases} p_1 = 0.005, & i=1, 2 \\ p_2 = 0.008, & i=3, 4 \end{cases}$$

$$\begin{aligned} P(\text{signal does not arrive}) &= P\{ (R_1 \text{ or } R_2 \text{ opens}) \text{ and } (R_3 \text{ or } R_4 \text{ opens}) \} \\ &= P\{ (R_1 \cup R_2) \cap (R_3 \cup R_4) \} = P(R_1 \cup R_2) P(R_3 \cup R_4) \quad (\text{independent} \\ &\quad \text{failures}) = [P(R_1) + P(R_2) - P(R_1 \cap R_2)] [P(R_3) + P(R_4) - P(R_3 \cap R_4)] \\ &= (2p_1 - p_1^2)(2p_2 - p_2^2) = [0.01 - 25(10^{-6})][0.016 - 64(10^{-6})] \approx 0.00016. \end{aligned}$$

1.5-6.

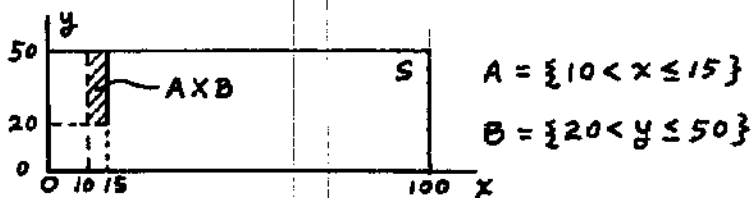
$$\begin{aligned} P\{ \text{signal does not arrive} \} &= P\{ (\text{upper path fails}) \text{ and } (\text{lower} \\ &\quad \text{path fails}) \} = P\{ (\text{upper path fails}) \cap (\text{lower path fails}) \} \\ &= P(\text{upper path fails}) P(\text{lower path fails}). \quad \text{But } P(\text{upper path fails}) \\ &= P(R_1) + P(R_2 \cap R_3) - P(R_1 \cap R_2 \cap R_3) = p_1 + p_2^2 - p_1 p_2^2 = 5.0995(10^{-3}), \text{ and} \\ P(\text{lower path fails}) &= p_2 + p_3^2 - p_2 p_3^2 = 12.475(10^{-3}). \quad \text{Thus,} \\ P\{ \text{signal does not arrive} \} &= 5.0995(10^{-3}) 12.475(10^{-3}) = 63.6163(10^{-6}). \end{aligned}$$

1.6-1. Let  $c_1, c_2, c_3, c_4$  and  $c_5$  represent the five cities and let  $m_1, m_2, m_3$  and  $m_4$  represent the motels. Then  $S_1 = \{c_1, c_2, c_3, c_4, c_5\}$  and  $S_2 = \{m_1, m_2, m_3, m_4\}$ . The combined sample space becomes

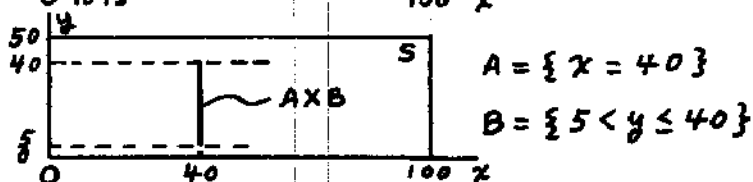
1.6-1. (Continued)

$$S = S_1 \times S_2 = \{(c_1, m_1), (c_1, m_2), (c_1, m_3), (c_1, m_4), \\ (c_2, m_1), (c_2, m_2), (c_2, m_3), (c_2, m_4), \\ (c_3, m_1), (c_3, m_2), (c_3, m_3), (c_3, m_4), \\ (c_4, m_1), (c_4, m_2), (c_4, m_3), (c_4, m_4), \\ (c_5, m_1), (c_5, m_2), (c_5, m_3), (c_5, m_4)\}.$$

1.6-2. (a)



(b)



1.6-3. Sequences =  $6(6)6(6) = 6^4 = 1296.$

1.6-4. Number of poker hands =  $\binom{52}{5} = \frac{52(51)50(49)48}{5(4)3(2)} = 2,598,960.$

1.7-1. This is a Bernoulli trials experiment with  $N=4,$

$p = P(\text{a can is out of tolerance}) = 0.03.$

(a)  $P(4 \text{ out of tolerance}) = \binom{4}{4} (0.03)^4 (1-0.03)^0 = 8.1(10^{-7}).$

(b)  $P(2 \text{ out of tolerance}) = \binom{4}{2} (0.03)^2 (1-0.03)^2$   
 $= \frac{4!}{2!2!} (9)10^{-4} (0.97)^2 \approx 5.081(10^{-3}).$

(c)  $P(\text{all in tolerance}) = P(\text{none is out of tolerance})$   
 $= \binom{4}{0} (0.03)^0 (1-0.03)^4 = (0.97)^4 \approx 0.8853.$

1.7-2. This is a Bernoulli trials experiment with  $N=6$ ,  
 $p = P(\text{land in recovery zone}) = 0.8$ .

$$(a) P(\text{none in zone}) = \binom{6}{0} (0.8)^0 (1-0.8)^6 = (0.2)^6 = 6.4(10^{-5}).$$

$$(b) P(\text{at least one in zone}) = 1 - P(\text{none in zone}) = 1 - 6.4(10^{-5}) = 0.999936.$$

$$(c) P(\text{success}) = P(3 \text{ in zone}) + P(4 \text{ in zone}) + P(5 \text{ in zone}) + P(6 \text{ in zone}) \\ = \binom{6}{3} (0.8)^3 (0.2)^3 + \binom{6}{4} (0.8)^4 (0.2)^2 + \binom{6}{5} (0.8)^5 (0.2)^1 + \binom{6}{6} (0.8)^6 (0.2)^0 \approx 0.983.$$

Yes, the program is successful.

1.7-3. For  $N=2$ :  $P\{\text{carrier sunk}\} = P\{2 \text{ hits}\} = \binom{2}{2} (0.4)^2 \cdot (1-0.4)^0 = 0.16$ . For  $N=4$ :  $P\{\text{carrier sunk}\} = P\{2 \text{ hits}\} + P\{3 \text{ hits}\} + P\{4 \text{ hits}\} = \binom{4}{2} (0.4)^2 (0.6)^2 + \binom{4}{3} (0.4)^3 (0.6)^1 + \binom{4}{4} (0.4)^4 (0.6)^0 = 0.5248$ .

1.7-4.  $P(\text{late } k \text{ of } N \text{ times}) = \binom{N}{k} 0.4^k (0.6)^{N-k}$ ,  $N=5$ .

$$(a) P(\text{late 3 or more times in one week}) = \binom{5}{3} 0.4^3 (0.6)^2 + \binom{5}{4} 0.4^4 (0.6)^1 + \binom{5}{5} 0.4^5 = 0.2304 + 0.0768 + 0.01024 = 0.31744.$$

$$(b) P(\text{not late at all}) = \binom{5}{0} 0.4^0 (0.6)^5 = 0.07776.$$

1.7-5. (a)  $P(0 \text{ late}) = \binom{5}{0} (0.3)^0 (0.7)^5 = 0.16807$ . (b)  $P(\text{all late}) = \binom{5}{5} 0.3^5 (0.7)^0 = 0.00243$ . (c)  $P(3 \text{ or more on time}) = P(0 \text{ late}) + P(1 \text{ late}) + P(2 \text{ late}) = 0.16807 + \binom{5}{1} 0.3 (0.7)^4 + \binom{5}{2} 0.3^2 (0.7)^3 = 0.83692$ .

1.7-6.  $P(\text{all on time}) = P(0 \text{ late}) = \binom{5}{0} x^0 (1-x)^5 = (1-x)^5$   
 $= 0.9$ . Thus,  $x = P(\text{a flight is late}) = 1 - 0.9^{1/5} = 0.020852$ .  
 Yes - to reduce 0.3 to 0.020852!

1.7-7. (a)  $P(\text{win}) = P(2 \text{ heads}) = \binom{5}{2} 0.7^2 (0.3)^3 = 0.13230$ . No!

(b)  $P(\text{win}) = P(4 \text{ heads}) + P(5 \text{ heads}) = \binom{5}{4} 0.7^4 (0.3)^1$   
 $+ \binom{5}{5} 0.7^5 (0.3)^0 = 0.36015 + 0.16807 = 0.52822$ . Yes!

\*1.7-8. (a)  $P(\text{win set}) = P(5 \text{ hits in } 6) + P(6 \text{ hits in } 6)$   
 $= \binom{6}{5} (0.8)^5 (0.2)^1 + \binom{6}{6} (0.8)^6 (0.2)^0 = 0.393216 +$   
 $0.262144 = 0.65536$ . (b) Second Bernoulli trial  
 problem.  $P(3 \text{ sets of } 3) = \binom{3}{3} (0.65536)^3 (1-0.65536)^0$   
 $= 0.28147$ .

1.7-9.

$P(\text{successful arrival}) = P\{\text{engine survives and navigation system survives}\} = P\{\text{engine survives}\} P\{\text{navigation system survives}\}$   
 $= [1 - P(\text{engine fails})][1 - P(\text{navigation system fails})] = 0.95(0.999)$   
 $= 0.94905$ .

1.7-10.

(a)  $P(\text{all operate}) = P(0 \text{ fail}) = \frac{6!}{0!(6-0)!} (0.06)^0 (0.94)^{6-0} = 0.6899$ .  
 (b)  $P(\text{all fail}) = P(6 \text{ fail}) = \frac{6!}{6!(6-6)!} (0.06)^6 (0.94)^{6-6} = 4.666(10^{-8})$ .  
 (c)  $P(1 \text{ fails}) = \frac{6!}{1!(6-1)!} (0.06)^1 (0.94)^{6-1} = 0.2642$ .

(1.7-11.)

$$\begin{aligned} \text{(a) } P(2 \text{ or more not repaired}) &= 1 - P(0 \text{ not repaired}) - P(1 \\ \text{not repaired}) &= 1 - \frac{8!}{0!(8-0)!} (0.1)^0 (0.9)^{8-0} - \frac{8!}{1!(8-1)!} (0.1)^1 (0.9)^{8-1} \\ &\approx 1 - 0.4305 - 0.3826 = 0.1869. \end{aligned}$$

(b)  $P(8 \text{ properly repaired}) = P(0 \text{ not repaired}) \approx 0.4305.$

(1.7-12.)

$$\begin{aligned} P(0 \text{ fail to return key}) &= \frac{(Np)^k e^{-Np}}{(k=0)!} = \frac{2.5^0 e^{-2.5}}{0!} = 0.0821. \\ P(1 \text{ fails to return key}) &= \frac{2.5^1 e^{-2.5}}{1!} = 0.2052. \\ P(2 \text{ fail to return keys}) &= \frac{2.5^2 e^{-2.5}}{2!} = 0.2565. \\ P(3 \text{ fail to return keys}) &= \frac{2.5^3 e^{-2.5}}{3!} = 0.2138. \\ P(\text{no more than 3 fail to return keys}) &= 0.0821 + 0.2052 + 0.2565 \\ &+ 0.2138 = 0.7576. \end{aligned}$$