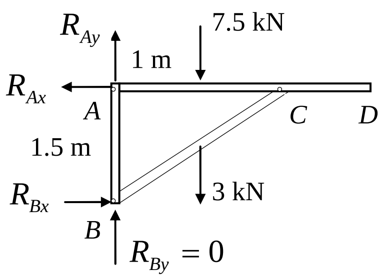
CHAPTER 1

SOLUTION (1.1)

(*a*)



Entire Structure

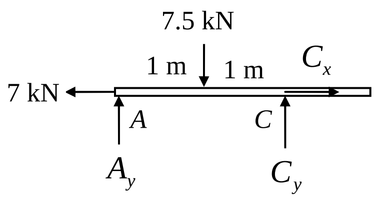
$$\sum M_A = 0: \quad 7.5(1) + 3(1) = 1.5 R_{Bx}$$

or $$R_{Bx} = 7\ kN$$

$$\sum F_x = 0: \quad R_{Ax} = -7\ kN$$ ◄

$$\sum F_y = 0: \quad R_{Ay} = 10.5\ kN$$

(*b*)



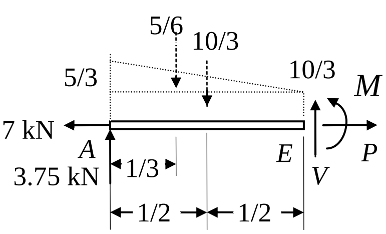
Member *AD*

$$\sum M_A = 0: \ C_y(2) = 7.5(1); \ C_y = 3.75\ kN$$

$$\sum F_y = 0: \ A_y = 7.5 - 3.75 = 3.75\ kN$$ ◄

$$\sum F_x = 0: \quad C_x = 7\ kN$$

(*c*)



Segment *AE*

$$\sum F_x = 0: \quad P = 7\ kN$$

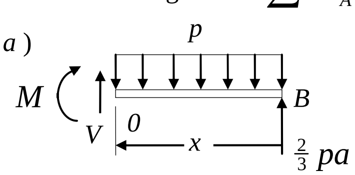
$$\sum F_y = 0: \quad V = 3.75 - \tfrac{5}{6} - \tfrac{10}{3} = 0.417\ kN$$

$$\sum M_E = 0: \quad M = 3.75(1) - \tfrac{5}{6}(\tfrac{2}{3}) - \tfrac{10}{3}(\tfrac{1}{2})$$
$$= 1.528\ kN \cdot m$$

SOLUTION (1.2)

Refer to Fig. P1.2: $\sum M_A = 0: \quad R_B = \frac{2}{3} pa \uparrow$

(*a*)



$$\sum M_0 = 0: \qquad M = \tfrac{2}{3} pax - \tfrac{1}{2} px^2 \qquad (\boldsymbol{a})$$ ◄

Thus

$$\tfrac{dM}{dx} = 0: \quad \tfrac{2}{3} pa - px = 0; \quad x = \tfrac{2}{3} a$$

Equation (***a***), for $x = \frac{2}{3} a$; $\quad M_{\max} = \frac{2}{9} pa^2$ ◄

(*b*) Equation (***a***), for $x = \frac{3}{2} a$:

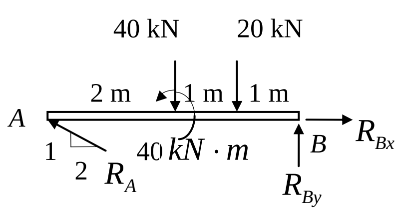
$$M = \tfrac{2}{3} pa(\tfrac{3}{2} a) - \tfrac{1}{2} p(\tfrac{3}{2} a)^2 = -\tfrac{1}{8} pa^2 = \tfrac{1}{8} pa^2 \circlearrowright$$ ◄

Shear force at $x = \frac{3}{2} a$:

$$V = -\tfrac{2}{3} pa + \tfrac{3}{2} pa = \tfrac{5}{6} pa \uparrow$$ ◄

SOLUTION (1.3)

(*a*)



Member *AB*

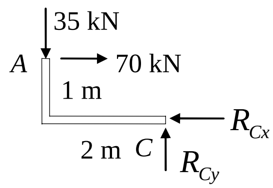
$$\sum M_B = 0: \quad R_A = 35\sqrt{5}\ kN$$

$$\sum F_x = 0: \quad R_{Bx} = 70\ kN$$ ◄

$$\sum F_y = 0: \quad R_{By} = 25\ kN$$

(CONT.)

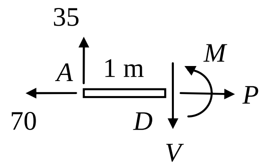
(1.3 CONT.)



Member *AC*

$$\sum M_A = 0: \quad R_{Cx} = 2R_{Cy}$$
$$\sum F_y = 0: \quad R_{Cy} = 35\ kN \quad ◄$$
$$\sum F_x = 0: \quad R_{Cx} = 70\ kN$$

(*b*)

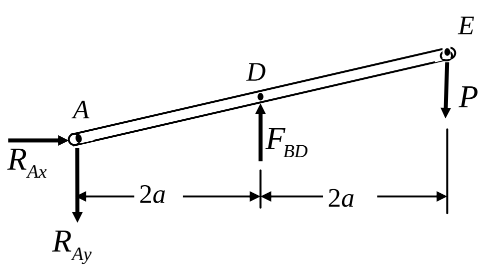


Segment *AD*

$$\sum F_x = 0: \quad P = 70\ kN$$
$$\sum F_y = 0: \quad V = 35\ kN \quad ◄$$
$$\sum M_D = 0: \quad M = 35\ kN \cdot m$$

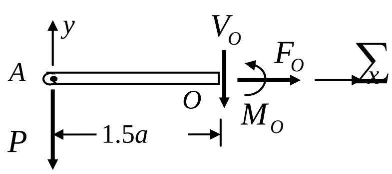
SOLUTION (1.4)

Link BD is a two-force member and hence the direction of F_{BD} is known.



(*a*) Free body-Member *ADE*

$$\sum M_A = 0:$$
$$-P(4a) + F_{BD}(2a) = 0, \quad F_{BD} = 2P \uparrow$$
$$\sum F_x = 0: \quad R_{Ax} = 0$$
$$\sum F_y = 0:$$
$$-R_{Ay} + F_{BD} - P = 0, \qquad R_{Ay} = P \downarrow$$



(*b*) Free Body-Part *AO*

$$\sum F_x = 0: \quad F_O = 0$$
$$\sum F_y = 0: \quad V_O = P \uparrow \quad ◄$$
$$\sum M_O = 0: \quad M_O = 1.5Pa \circlearrowright$$

SOLUTION (1.5)

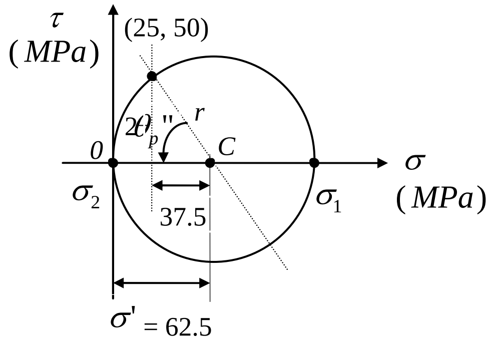
We have $\theta = 115^o$. Apply Eqs. (1.11):

$$\sigma_{xn} = \tfrac{1}{2}(-50+40) + \tfrac{1}{2}(-50-40)\cos 230^o - 20\sin 230^o$$
$$= -5 + 28.92 + 15.32 = 39.2 MPa \quad ◄$$

$$\tau_{x'y'} = -\tfrac{1}{2}(-50-40)\sin 230^o - 20\cos 230^o = -21.6\ MPa \quad ◄$$

SOLUTION (1.6)

(*a*)



$$\theta_p'' = \tfrac{1}{2}\tan^{-1}\tfrac{50}{37.5} = 26.6^o$$

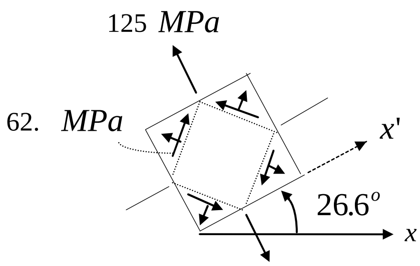
$$r = (50^2 + 37.5^2)^{1/2} = 62.5$$

Thus,

$$\sigma_1 = 62.5 + 62.5 = 125 \ MPa$$

$$\sigma_2 = 0$$

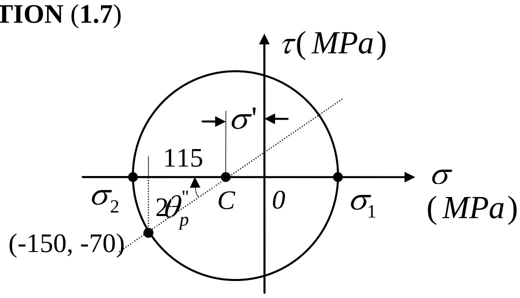
(*b*)



$$\tau_{\max} = r = 62.5 \ MPa$$

SOLUTION (1.7)

(*a*)



$$\sigma' = \tfrac{1}{2}(-150 + 80) = -35 \ MPa$$

$$\theta_p'' = \tfrac{1}{2}\tan^{-1}\tfrac{70}{115} = 15.66^o$$

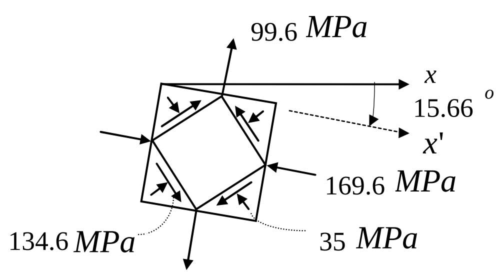
$$r = (115^2 + 70^2)^{1/2} = 134.6$$

Thus,

$$\sigma_1 = -35 + 134.6 = 99.6 \ MPa$$

$$\sigma_2 = -169.6 \ MPa$$

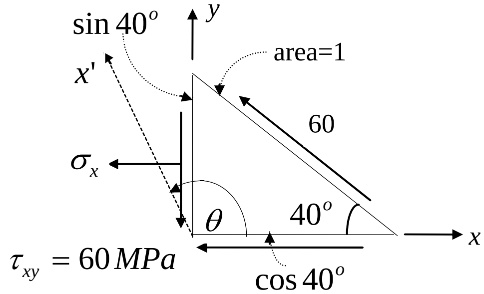
(*b*)

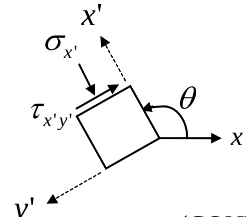


$$\tau_{\max} = r = 134.6 \ MPa$$

SOLUTION (1.8)

(*a*)





(CONT.)

(1.8 CONT.)

We have $\sigma_y = 0$ and $\tau_{xy} = 60\,MPa$.

$$\sum F_x = 0: \qquad \sigma_x \sin 40^o = 60\cos 40^o + 60\cos 40^o$$

$$\text{or} \qquad \sigma_x = 143\ MPa\ (comp.)$$

Apply Eqs. (1.11) with $\theta = 90 + 25 = 115^o$:

$$\sigma_{x'} = -\tfrac{143}{2} - \tfrac{143}{2}\cos 230^o + 60\sin 230^o = -71.5\ MPa$$

$$\tau_{x'y'} = 71.5\sin 230^o + 60\cos 230^o = -93.34\ MPa$$

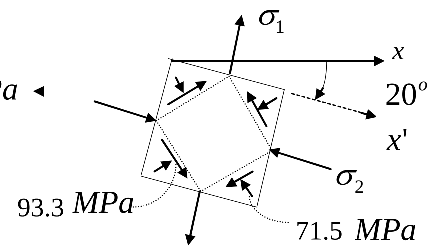
(*b*)

$$\tau_{\max} = [(-71.5)^2 + 60^2]^{1/2} = 93.34\ MPa$$

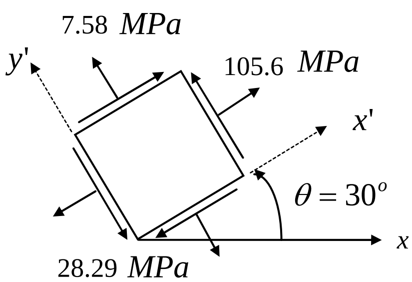
$$\sigma' = -71.5\ MPa$$

It may be seen from a sketch of Mohr's Circle that

$$\theta_p'' = \tfrac{1}{2}\tan^{-1}\tfrac{60}{71.5} = 20^o$$



SOLUTION (1.9)



$$\sigma_x = \sigma_y = 80\cos 45^o = 56.57\ MPa$$

$$\tau_{xy} = 80\sin 45^o = 56.57\ MPa$$

Apply Eqs. (1.11):

$$\sigma_{x'} = \tfrac{1}{2}(56.57 + 56.57) + 0 + 56.57\sin 60^o = 105.6\ MPa$$

$$\sigma_{y'} = 56.57 - 48.99 = 7.58\ MPa$$

$$\tau_{x'y'} = -0 + 56.57\cos 60^o = 28.29\ MPa$$

SOLUTION (1.10)

We have $\sigma_x = 0$, $\sigma_y = -50\ MPa$, $\tau_{xy} = 100\ MPa$, $\theta = 70^o$

Apply Eqs. (1.11):

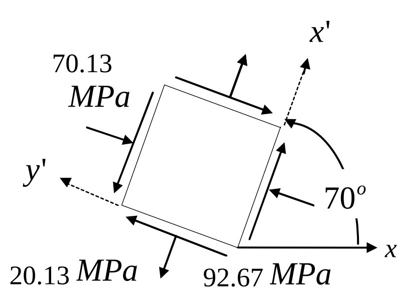
$$\sigma_{x'} = -\tfrac{50}{2} + \tfrac{50}{2}\cos 140^o + 100\sin 140^o$$

$$= 20.13\ MPa$$

$$\sigma_{y'} = -25 + 19.15 - 64.28 = -70.13\ MPa$$

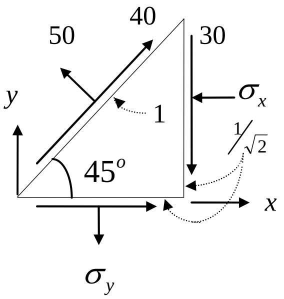
$$\tau_{x'y'} = -25\sin 140^o + 100\cos 140^o$$

$$= -92.67\ MPa$$



SOLUTION (1.11)

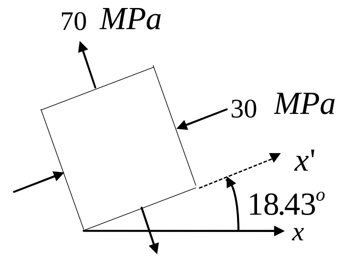
(*a*)



$$\tau_{xy} = -30\ MPa\,,\quad \sigma_y = 60\ MPa$$ ◄

$$\sum F_x = 0:\quad \sigma_x(\tfrac{1}{\sqrt{2}}) = 30(\tfrac{1}{\sqrt{2}}) + 40(\tfrac{1}{\sqrt{2}}) - 50(\tfrac{1}{\sqrt{2}})$$

or $\sigma_x = 20\ MPa\ \ (comp.)$ ◄



(*b*) It may be seen from a sketch of Mohr's circle that

$$\theta_p'' = \tfrac{1}{2}\tan^{-1}\tfrac{3}{4} = 18.43^o$$

$$\sigma_{1,2} = \tfrac{-20+60}{2} \pm [(\tfrac{-20-60}{2})^2 + (-30)^2]^{1/2}$$

$$= 20 \pm 50$$

or $\sigma_1 = 70\ MPa\,,\ \ \sigma_2 = -30\ MPa$ ◄

SOLUTION (1.12)

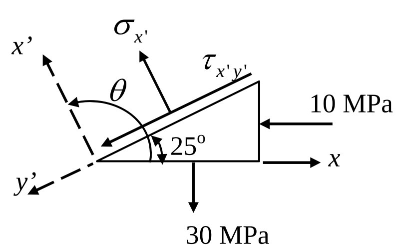
We have

$$\theta = 25 + 90 = 115^o$$

$$\sigma_x = -10\ MPa$$

$$\sigma_y = 30\ MPa$$

$$\tau_{xy} = 0$$



(*a*) $\sigma_{x'} = \tfrac{1}{2}(\sigma_x + \sigma_y) + \tfrac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta$

$$= \tfrac{1}{2}(-10+30) + \tfrac{1}{2}(-10-30)\cos 230^o = 22.86\ MPa$$ ◄

Thus,

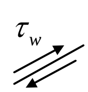
$$\sigma_w = \sigma_{x'} = 22.86\ MPa$$

(*b*) $\tau_{x'y'} = -\tfrac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta$

$$= -\tfrac{1}{2}(-10-30)\sin 230^o = -15.32\ MPa$$

So

$$\tau_w = \tau_{x'y'} = -15.32\ MPa$$ ◄



SOLUTION (1.13)

(*a*) $\varepsilon_x = \alpha\,\Delta T = 11.7(10^{-6})(80^o) = 936\ \mu$

Thus $\delta = \varepsilon_x L = 936(10^{-6})250 = 0.234\ mm\ \longrightarrow$ ◄

(*b*) $\varepsilon_x = \alpha\,\Delta T = 11.7(10^{-6})(\tfrac{80x}{L}) = 9.36(10^{-4})\tfrac{x}{L}$

Hence $u = \int \varepsilon_x dx = 4.68(10^{-4})\tfrac{x^2}{L} + c$ (***a***)

We have u=0 at x=0; c=0. Equation (***a***) for x=250 mm gives $\delta = 0.117\ mm \longrightarrow$ ◄

SOLUTION (1.14)

From solution of Prob. 1.11:

$$\sigma_x = -20\ MPa\,,\quad \sigma_y = 60\ MPa\,,\quad \tau_{xy} = -30\ MPa$$

Thus, $\varepsilon_x = \frac{1}{70(10^3)}[-20 - 0.3(60)] = -543\ \mu\,,\quad \varepsilon_y = \frac{1}{70(10^3)}[60 + 0.3(20)] = 943\ \mu$ ◄

$$\gamma_{xy} = -\frac{30}{70(10^3)/2.6} = -1114\ \mu$$

(*a*) Equations (1.25) with $\theta = 180 - 60 = 120^o$:

$$\varepsilon_{x'} = \frac{-543+943}{2} + \frac{-543-943}{2}\cos 240^o - 1114\sin 240^o$$

$$= 200 + 371.5 + 964.7 = 1536\ \mu$$ ◄

(*b*) $\gamma_{\max} = 2[(\frac{-543-943}{2})^2 + (-\frac{1114}{2})^2]^{1/2} = 1857\ \mu$ ◄

SOLUTION (1.15)

(*a*) $\varepsilon_{1,2} = \frac{50+250}{2} \pm [(\frac{50-250}{2})^2 + (-\frac{150}{2})]^{1/2} = 150 \pm 125$

or $\varepsilon_1 = 275\ \ \mu\,,\quad \varepsilon_2 = 25\ \ \mu$ ◄

(*b*) Apply Hooke's Law with $\sigma_z = 0$:

$$275(10^{-6}) = \frac{1}{210(10^9)}(\sigma_1 - 0.3\sigma_2);\qquad 57.75 = \sigma_1 - 0.3\sigma_2$$

$$25(10^{-6}) = \frac{1}{210(10^9)}(\sigma_2 - 0.3\sigma_1);\qquad 5.25 = \sigma_2 - 0.3\sigma_1$$

Solving, $\sigma_1 = 65.19\ MPa\,,\quad \sigma_2 = 24.81\ MPa$ ◄

SOLUTION (1.16)

We have

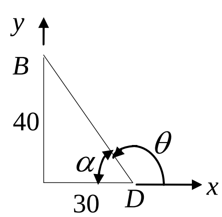
(*a*) $\varepsilon_x = \frac{0.006}{50} = 120\ \mu \qquad \varepsilon_y = -\frac{0.004}{25} = -160\ \mu$

$$\gamma_{xy} = -1000 - 50 = -1500\ \mu\,\text{rad}$$ ◄

(*b*) $\gamma_{\max} = 2\mu[(\frac{120+160}{2})^2 + (-\frac{1500}{2})^2]^{1/2} = 1526\ \mu$ ◄

SOLUTION (1.17)

(*a*)



$$\alpha = \tan^{-1}\tfrac{4}{3} = 53.13^o,$$

$$\theta = 126.87^o$$

Apply Eq. (1.25);

$\varepsilon_{x'} = \frac{400+800}{2} + \frac{400-800}{2}\cos 2(126.87^o) + \frac{200}{2}\sin 2(126.87^o) = 560\ \mu$ ◄

(*b*) $\varepsilon_{1,2} = \frac{400+800}{2} \pm [(\frac{400-800}{2})^2 + (\frac{200}{2})^2]^{1/2} = 600 \pm 224$

or $\varepsilon_1 = 824\ \mu \qquad \varepsilon_2 = 376\ \mu$ ◄

It may be seen from a sketch of Mohr's Circle that;

$$\theta_p'' = \tfrac{1}{2}\tan^{-1}(\tfrac{1}{2}) = 13.28^o$$

SOLUTION (1.18)

(*a*) $\varepsilon_x = \frac{1}{100(10^3)}(150 + \frac{90}{3}) = 1800\ \mu, \quad \varepsilon_y = \frac{1}{100(10^3)}(-90 - \frac{150}{3}) = -1400\ \mu$

$\varepsilon_z = -\frac{1/3}{100(10^3)}(150 - 90) = -200\ \mu$

$\Delta a = 1800\mu(100) = 180\ \mu m, \quad \Delta b = -1400\mu(50) = -70\ \mu m,$

$\Delta t = -200\mu(10) = -2\ \mu m$

Thus,

$$a' = 1000.18\ mm, \quad b' = 49.93\ mm, \quad t' = 9.998\ mm$$ ◂

(*b*) $A'C' = [100.8^2 + 49.93^2]^{1/2} = 111.933\ mm$ ◂

SOLUTION (1.19)

(*a*) We have $\sigma_x = \sigma_y = \sigma_z = -p$, AC =111.8034 mm.

Equation (1.34),

$$\varepsilon_x = \varepsilon_y = \varepsilon_z = -\frac{P}{E}(1 - 2\nu) = -\frac{120}{100(10^3)}(\frac{1}{3}) = -400\ \mu \qquad (\boldsymbol{a})$$

and

$\Delta a = -400\mu(100) = -40\ \mu m, \quad \Delta b = -400\mu(50) = -20\ \mu m$

$\Delta t = -400\mu(10) = -4\ \mu m$

Hence

$$a' = 99.96\ mm, \quad b' = 49.98\ mm, \quad t' = 9.996\ mm$$ ◂

(*b*) $A'C' = [99.96^2 + 49.98^2]^{1/2} = 111.7587\ mm$ ◂

SOLUTION (1.20)

$\angle DAC = \theta = \tan^{-1}\frac{3}{4} = 36.9^o$

$L_{AC} = L_{BD} = \sqrt{90^2 + 120^2} = 150\ mm \qquad G = \frac{E}{2(1+1/3)} = \frac{3}{8}E$

$\varepsilon_x = \frac{1}{E}(150 + \frac{75}{3}) = \frac{175(10^6)}{E}$

$\varepsilon_y = \frac{1}{E}(75 + \frac{150}{3}) = \frac{125(10^6)}{E}$

$\gamma_{xy} = \frac{150}{G} = \frac{400(10^6)}{3E}$

We have

$\theta_1 = 180 - 36.9 = 143.1^o$

Taking x' along BD

$\varepsilon_{x'} = (25 + 150\cos 286.2^o + 66.67\sin 286.2^o)\frac{10^6}{E}$

$= (25 + 41.85 - 64)\frac{10^6}{E} = 2.85(10^6)/E$

$\Delta L_{BD} = 100\frac{2.85(10^6)}{210(10^9)} = 1.35\times 10^{-3}\ mm$ ◂