1.1 Substitute (1.2-2) into (1.2-1):

\[ u_1 = \beta_1 \]

hence \( \beta_1 = u_1 + u_2 - u_1 \)

\[ u_2 = \beta_1 + \beta_2 a = u_1 + \beta_2 a \]

hence \( \beta_2 = \frac{a}{b} \)

\[ u_3 = \beta_1 + \beta_3 b = u_1 + \beta_3 b \]

hence \( \beta_3 = \frac{u_3 - u_1}{b} \)

(1.2-1a) becomes \( u = u_1 + \frac{u_2 - u_1}{a} x + \frac{u_3 - u_1}{b} y \)

or \( u = \left(1 - \frac{x}{a} - \frac{y}{b}\right) u_1 + \frac{x}{a} u_2 + \frac{y}{b} u_3 \)

Similarly for \( v \)

1.2 On side 1-2, \( y = 0 \), sub. into (1.2-3b):

\[ v = \left(1 - \frac{x}{a}\right) v_1 + \frac{x}{a} v_2 \]

Depends only on \( v_1 \) and \( v_2 \) and is a linear function of \( x \), \( i.e \), 1-2 remains straight.

\[ s \]

On side 2-3:

\[ x = a - s \sin \phi = a - \frac{a}{h} s \]

\[ y = s \cos \phi = \frac{b}{h} s \]

(1.2-3a) becomes

\[ u = \left(1 - \frac{s}{h}\right) u_1 + \left(1 - \frac{s}{h}\right) u_2 + \frac{h}{s} u_3 \]

\[ u = \left(1 - \frac{s}{h}\right) u_2 + \frac{h}{s} u_3 \]

Similarly for \( v \)

Now \( u \) & \( v \) along 2-3 depend only on \( u_2, v_2, u_3, v_3 \) vary linearly with \( s \), \( i.e \), edge remains straight.

1.3 In all parts, substitute coords. into (1.2-1a); \( v \) equation is similar.

\[ u_1 = \beta_1 + \beta_2 a \quad \text{A) } \]

\[ u_2 = \beta_1 + \beta_2 a + \beta_3 b \quad \text{B) } \]

\[ u_3 = \beta_1 + \beta_3 b \quad \text{C) } \]

(B) & (C) yield \( \beta_1 = u_1 - u_2 + u_3 \quad \text{D) } \)

(A) & (D) yield \( \beta_2 = \frac{u_2 - u_3}{a} \)

(1.2-1a) becomes

\[ u = (u_1 - u_2 + u_3) + \frac{u_2 - u_3}{a} x + \frac{u_3 - u_1}{b} y \]

\[ u = \left(1 - \frac{y}{b}\right) u_1 + \left(1 - \frac{x}{a} + \frac{y}{b}\right) u_2 + \left(1 - \frac{x}{a}\right) u_3 \]

Likewise \( v = \left(1 - \frac{s}{h}\right) v_1 + \frac{h}{s} v_3 \)

Conclusion: as for part (b).
1.5

(a) \( \varepsilon_x = \frac{\partial u}{\partial x} = \beta_2 + 2\beta_4 + \beta_8 y \)
\( \varepsilon_y = \frac{\partial v}{\partial y} = \beta_3 + \beta_9 x + \beta_6 y + \frac{\partial^2 u}{\partial y^2} \)
\( \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \beta_3 + \beta_4 + (\beta_5 + 2\beta_{10}) \frac{x}{x} + (2\beta_6 + \beta_9) y \)

(b) To permit y-direction displacement associated with the Poisson effect, if all supports at \( x = 0 \) were pins, we would not obtain \( \sigma_y = 0 \), as beam theory says.

(c) (there is no relative y displacement between top and bottom surfaces)

(d) According to beam theory, \( \varepsilon_x = c y \), \( \varepsilon_y = 2 \varepsilon_x \), and \( \gamma_{xy} = 0 \), where \( c = \text{const} \).
Therefore \( \beta_2 = \beta_4 = \beta_8 = \beta_9 = 0 \),
and from the \( \gamma_{xy} \) expression \( \beta_3 + \beta_4 = 0 \),
Also, from \( \gamma_{xy} \) \( \beta_6 = 0 \) and \( \beta_5 + 2\beta_{10} = 0 \).

1.6

Should approach with mesh refinements.

\( \sigma_x = 0 \) along AB, CD, and at E.
\( \sigma_y = 0 \) along BC and the straight part of ED
\( \tau_{xy} = 0 \) along all of the boundary except the arc of radius \( r \)

\( \sigma_y \) and \( \tau_{xy} \) very large & negative at C
\( \sigma_y \) large and negative at E.

1.7

Approximation:

\[ r = r_n \left( \frac{\text{d}A}{\text{d}r} = \frac{t \ln \frac{12.315}{6}}{1.2993t} = 0.719t \right) \]

Largest \( \sigma_r \) is almost at the neutral axis, let's evaluate at the neutral axis.

\[ \sigma_r = M e_t r_n \left( r_n \text{d}A - A_r \right) \]

\[ \sigma_r = \frac{-20P}{16t(1.685)t(12.315)} \left( 12.315 \left( 0.719t \right) - 6.315t \right) \]

\( \sigma_r = -0.153 \frac{P}{t} \)

Using theory & formulas of Ref. 2.1,

\[ \Theta_F = \frac{PR^2}{EI} + \frac{\pi}{2} \frac{M_F R^2}{EI} \]

\[ V_F = \frac{\pi}{4} \frac{PR^3}{EI} + \frac{M_F R^2}{EI} \]
\[ V_d = V_F + \Theta_p L + \frac{PL^3}{3EI_s} \quad \Gamma_x = \frac{4}{12} \frac{12}{3} \]

\[ V_d = \frac{PR^2}{EI} \left( \frac{\pi R}{4} + L + L + \frac{\pi R^2}{2R} \right) + \frac{PL^3}{3EI_s} \]

\[ V_d = \frac{P(14L^2)}{341t^2} \left( \frac{\pi}{4} + L + \frac{\pi R^2}{2R} \right) + \frac{P(t)}{3E(t)} = \frac{160P}{1t} \text{ (down)} \]

\[ \Delta_3 = 0.4554 \frac{PL_T}{E_0 h_t} = -1.5 \%
\]

(c) \( \frac{0}{x} \) \( \frac{ex}{A} = -\frac{P}{E} \left( \frac{3h_t}{L_t} \right) = -\frac{P}{E h_t (1 + \frac{3x}{L_t})} \)

\[ u = \beta_1 + \beta_2 x \quad (a) \quad \text{(form of Eq. 1.2-1)} \]

To obtain form of Eq. 1.2-3, evaluate (a) at \( x = 0 \) and at \( x = L \).

\[ x = 0: \quad u_1 = \beta_1 \]
\[ x = L: \quad u_2 = \beta_1 + \beta_2 L \]

\[ \beta_1 = u_1 \]
\[ \beta_2 = \frac{u_2 - u_1}{L} \]

Eq. (a) becomes \( u = u_1 + \frac{u_2 - u_1}{L} x \) or \( u = (1 - \frac{x}{L}) u_1 + \frac{x}{L} u_2 \)

1.9

\[ h_o, 1.75h_o, 2.5h_o, 3.25h_o \]

\[ \frac{4h_o}{L_t + d} = \frac{h_o}{d}, \quad d = \frac{L_t}{3} \]

\[ \frac{h_s}{h_o} = \frac{h_o}{d}, \quad h = \frac{h_o}{s} \]

\[ \Delta_{e_x} = \int_{d}^{d + L_t} \frac{P}{E h_o} ds = \int_{d}^{d + L_t} \frac{P}{E h_o} ds = \frac{4L_t^3}{3Eh_o} \]

\[ \Delta_{e_x} = \frac{PL_T}{3Eh_o} \ln 4 = 0.4621 \frac{PL_T}{Eh_o} \]

(b) \[ \Delta_1 = \frac{PL_T}{(2.5th_o)E} = 0.4 \frac{PL_T}{th_o E} = -13.1\% \]

\[ \Delta_2 = \frac{PL_T}{(1.75th_o)E} + \frac{PL_T}{(3.25th_o)E} = 0.4396 \frac{PL_T}{Eh_o} = -4.9\% \]

We see that when elements are small enough for the curve to be almost a straight line over \( L \), stress error is halved when no. of els. is doubled.