Mathematics

1. \[ \sum_{j=1}^{5} (j+1)^2 - 1 = (1+1)^2 - 1 + (2+1)^2 - 1 + (3+1)^2 - 1 + (4+1)^2 - 1 + (5+1)^2 - 1 = 2^2 + 3^2 + 4^2 + 5^2 + 6^2 - 5 = 85 \]

2. \[ y(2.7) = 7(2.7) + 4.2 = 11.756 \]
   \[ y(3.2) = 7(3.2) + 4.2 = 9.916 \]
   \[ y(3.9) = 7(3.9) + 4.2 = 12.534 \]
   
   The estimated value is \[ 9.916 + 0.7(12.534 - 9.916) = 11.749 \]
   
   The error is \[ \frac{11.756 - 11.749}{11.756} = 0.006 \text{ or } 0.06\% \]

3. Let \( d \) be the diameter.
   \[ V_{sphere} = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi \left( \frac{d}{2} \right)^3 = 0.524 d^3 \]
   \[ V_{cone} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \left( \frac{d}{2} \right)^2 h = 0.262 d^3 h \]
   
   Out \[ 0.524 d^3 = 0.262 d^3 h \]
   \[ h = 2.00 d \]

4. \[ \sin 20^\circ = 0.34 \]
   \[ n = 5 \cos 20^\circ = 4.7 \]

5. Expand by 2nd column
   \[ = -2 \begin{vmatrix} 4 & 3 \\ 5 & 5 \end{vmatrix} = -2(20 - 27) = 14 \]

6. From Eqn 6.3
   \[ 250^\circ + 460^\circ = 710^\circ R \]
   \[ \frac{5}{9}(250 - 32) = 121.1^\circ C \]

7. From page 1-42,
   \[ K = 1.71 \text{ EE-9} \]
   \[ Btu = \frac{1.71 \text{ EE-9} \text{ FT}^2 \cdot \text{HR} \cdot \text{R}^4}{(1.71 \text{ EE-9} \text{ FT} \cdot \text{HR} \cdot \text{R}^4) \left( \frac{17.51}{\text{WATT-MIN}} \cdot \frac{1}{\text{Btu}} \right) \left( \frac{1}{60 \text{ MIN}} \right)} \]
   \[ (0.3648 \text{ m}^3 \text{A}) \left( \frac{54 \text{ Kf} \cdot \text{R}}{\text{m}^3 \cdot \text{Kf}} \right)^4 \]
   \[ = 5.66 \text{ EE-8 WATTS} \]

8. \[ y = 6 + 0.75(2-6) = 3.0 \]

9. The slope is \[ \frac{9.5-3.4}{8.3-1.7} = 0.924 \]
   Using the first point,
   \[ (y-3.4) = 0.924(x-1.7) \]

10. Let \( x \) be the number of elapsed periods of 0.1 second. Let \( y \) be the amount present after \( x \) periods
    \[ Y_1 = 1.001 Y_0 \]
    \[ Y_2 = (1.001)^2 Y_0 \]
    \[ Y_n = (1.001)^n Y_0 \]

    Now \[ \frac{Y_n}{Y_0} = 2 = (1.001)^n \]

    \[ \log(2) = n \log(1.001) \]
    \[ n = \frac{\log(2)}{\log(1.001)} \]
    \[ n = 693.5 \text{ periods} \]
    \[ t = 69.35 \text{ seconds} \]

**Concentrates**

**First, rearrange**

\[ X + Y = -4 \]
\[ X + Z = 1 \]
\[ 3X - Y + 2Z = 4 \]

Now use Cramer's rule (page 1-6)

\[ \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 3 & -1 & 2 \end{vmatrix} = 11z = 2 \]

\[ \begin{vmatrix} 1 & -4 & 0 \\ 0 & -1 & -6 \\ 4 & 1 & 3 \end{vmatrix} = 11z = 4 \]

\[ \begin{vmatrix} -4 & -2 & -1 \\ 1 & 1 & 1 \\ 4 & 1 & 3 \end{vmatrix} = x = \frac{4}{5} \]

\[ \begin{vmatrix} -1 & -6 & 1 \\ 0 & -6 & 1 \\ 4 & 3 & 2 \end{vmatrix} = y = \frac{3}{2} \]

\[ \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 3 & -1 & 4 \end{vmatrix} = z = \frac{4}{5} \]
Always graph the data first to see if it is a straight line. In this case, it is.

\[ y = \frac{1}{x} \]

Use linear regression (page 1-13)

\[ N = 7 \]
\[ \Sigma x = 12550 \]
\[ \bar{x} = 1792.9 \]
\[ \Sigma x^2 = 3,117,667 \]

\[ (\Sigma x)^2 = 1,575,888 \]

\[ (\Sigma x^2) = 3,117,667 \]

\[ \Sigma y = 12300 \]
\[ \bar{y} = 1757.1 \]
\[ \Sigma y^2 = 3,017,667 \]
\[ (\Sigma y)^2 = 1,513,667 \]

\[ \Sigma xy = 3,067,667 \]

\[ M = \frac{7(3,067,667) - (12550)(12300)}{7(3,117,667) - (12550)^2} = .994 \]

\[ b = 1757.1 - .994(1792.9) = -25.0 \]

so \[ y \approx .994x - 25.0 \]

The correlation coefficient is

\[ r = \frac{7(3,067,667) - (12550)(12300)}{\sqrt{(7)(3,117,667) - (12550)^2}(7)(3,017,667) - (12300)^2}} = .999 \]

This is a first-order linear differential equation. (page 1-30)

\[ m = \exp \int (-1) dx = e^{-x} \]
\[ y = e^x [2xe^{-x} + c] \]
\[ = e^x [2xe^{-x} - e^x + c] \]

But \[ y = 1 \] when \( x = 0 \)

\[ 1 = 1[0 - 2 + c] \]

\[ c = 3 \]

so \[ y = 2e^x(x - 1) + 3e^x \]

Upon graphing the data, we see that it is not a straight line.

It looks like an exponential with form

\[ t = b e^{ms} \]

Or perhaps

\[ \log t = b + ms \]

Try making the variable transformation

\[ R = \log t \]

<table>
<thead>
<tr>
<th>S</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>1.633</td>
</tr>
<tr>
<td>1.8</td>
<td>2.194</td>
</tr>
<tr>
<td>1.6</td>
<td>2.585</td>
</tr>
<tr>
<td>1.4</td>
<td>3.041</td>
</tr>
</tbody>
</table>

\[ N = 4 \]
\[ \Sigma S = 6.8 \]
\[ \bar{S} = 1.7 \]
\[ \Sigma S^2 = 117.6 \]
\[ (\Sigma S)^2 = 46.24 \]

\[ (\Sigma S^2) = 68.51 \]

\[ \Sigma SR = 155.28 \]

\[ M = \frac{4(155.28) - (68)(9.96)}{4(117.6) - (68)^2} = -.2328 \]

\[ b = 2.352 + .2328(17) = 6.3096 \]

so \[ R = 6.3096 - .2328S \]

or \[ \log t = 6.3096 - .2328S \]
6. Let \( x_e \) = pounds of salt in tank at time \( t \) 
\[ x_e = 60 \]

\( x' \) = rate at which salt content changes
2. Pounds of salt entering each minute
3. Gallons leaving each minute.

The salt leaving each minute is
\[ 3 \left( \frac{\text{concentration}}{\text{lb/gal}} \right) = 3 \left( \frac{\text{salt content}}{\text{volume}} \right) = 3 \left( \frac{x}{100-t} \right) \]

\[ x' = 2 - 3 \left( \frac{x}{100-t} \right) \]

Or
\[ x' + \frac{3x}{100-t} = 2 \]

This is first order linear (page 1-31)

\[ u = \exp \left[ 3 \int \frac{dt}{100-t} \right] = (100-t)^{-3} \]

\[ x = (100-t)^3 \left[ 2 \int \frac{dt}{(100-t)^3} + k \right] \]

\[ = 100-t + k (100-t)^3 \]

But \( x = 60 \) at \( t = 0 \),
so \( k = -0.00004 \)

\[ x = 100-t - 0.00004 (100-t)^3 \]

\[ x_{60} = 37.44 \text{ pounds} \]

7. If \( c \) is positive, then \( N(\infty) > 0 \),
which is contrary to the given data.
so \( c \leq 0 \).

If \( c = 0 \), then \( N(\infty) = \frac{a}{1+b} = 100 \)
which is possible dependent on \( a, b \).

If \( c > 0 \), then \( N(0) = \frac{a}{1+b} = 10 \) which conflicts with the previous step.

So \( c < 0 \), then \( N(\infty) = 0 \), so \( a = 100 \).
Now \( N(0) = \frac{a}{1+b} = \frac{100}{1+b} = 10 \), so \( b = 9 \).

\[ \frac{dN}{dt} = -100(1+9e^{-2}) (9) e^{-2} (c) \]

If \( t = 0 \), then \( c = -0.0556 \).

8. \[ \frac{dy}{dx} = 3x^2 - 18y \]

\[ 3x^2 - 18y = 0 \text{ at all extreme points} \]

\[ x^2 - 6x = 0 \text{ at } x = 0, x = 6 \]

\[ \frac{dy}{dx} = 6x - 18 \]

\[ 6x - 18 = 0 \text{ at inflection points} \]

\( x = 3 \) is an inflection point

\[ E = \text{MC}^2 = (0.001) \text{Kg} \cdot (3 \text{ EE8})^2 (\text{m/s})^2 \]

\[ = 9 \text{ EE}13 \text{ Joules} \]

\[ (9 \text{ EE}13) \int \frac{1}{2} \text{kJ} \cdot (9.476) \text{ BTU} \cdot \text{kJ} \]

\[ = 8.53 \text{ EE}10 \text{ BTU} \]

\[ \text{Tons} = \frac{6.53 \text{ EE}10 \text{ BTU}} {13 \text{ EE}0 \text{ BTU} \cdot (2000) \frac{40}{\text{ TON}}} \]

\[ = 3281 \text{ TONS} \]

9. The entropy contained in one gram of any substance is

\[ E = \text{MC}^2 = (0.001) \text{Kg} \cdot (3 \text{ EE8})^2 (\text{m/s})^2 \]

\[ = 9 \text{ EE}13 \text{ Joules} \]

\[ (9 \text{ EE}13) \int \frac{1}{2} \text{kJ} \cdot (9.476) \text{ BTU} \cdot \text{kJ} \]

\[ = 8.53 \text{ EE}10 \text{ BTU} \]

\[ \text{Tons} = \frac{6.53 \text{ EE}10 \text{ BTU}} {13 \text{ EE}0 \text{ BTU} \cdot (2000) \frac{40}{\text{ TON}}} \]

\[ = 3281 \text{ TONS} \]

10. The standard normal variables are

\[ z_1 = \frac{.502 - .497}{.005} = 1 \]

\[ z_2 = \frac{.507 - .502}{.005} = 1 \]

a) \[ P\{ \text{defective} \} = 2 \left( \frac{.5 - .3413}{2} \right) = .3174 \]

b) \[ P\{ 3.2 \} = \frac{3}{(3-2)} \cdot (\cdot3174)^2 (1 - .3174)^1 \]

\[ = .2063 \]

c) \[ (8)(200)(.3174) = 507.8 \]

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b) Use the cumulative distribution graph. For 75% (.75), the cell mid-point is approximately 40.

g) Use the cumulative graph to find the mid-point for 50%. This occurs at approximately 33.

\( \sum x_i = 1390 \), so mean = \( \frac{1390}{40} = 34.875 \)

h) \( \sum x^2 = 50496 \)

\[ \sigma = \sqrt{\frac{50496}{40} - \left(\frac{1390}{40}\right)^2} = 7.405 \]

i) \[ s = \sqrt{\frac{N}{N-1} (\sigma^2)} = \sqrt{\frac{40}{39}} (7.405) = 7.500 \]

j) \( s^2 = 56.27 \)

12) No contract deadline was given, so assume 36 as a scheduled time. Look for a path where (15-65) = 0 everywhere.

\[
\begin{array}{cccccc}
\text{START} & \text{A} & \text{B} & \text{C} & \text{D} & \text{E} \\
\text{0} & \text{0} & \text{0} & \text{7} & \text{7} & \text{7} \\
\text{7} & \text{9} & \text{13} & \text{7} & \text{13} & \text{7} \\
\text{12} & \text{16} & \text{13} & \text{7} & \text{13} & \text{7} \\
\text{12} & \text{25} & \text{29} & \text{16} & \text{29} & \text{13} \\
\text{31} & \text{36} & \text{36} & \text{36} & \text{36} & \text{36} \\
\text{36} & \text{36} & \text{36} & \text{36} & \text{36} & \text{36} \\
\text{FINISH} & & & & & \\
\end{array}
\]

c) 36

d) 36

e) 0

8) Float is same as slack = 0
To solve this as a regular CPM problem, it is necessary to calculate \( \bar{t}_{\text{mean}} \) and \( \sigma \) for each activity. For activity A,

\[
\bar{t}_{\text{mean}} = \frac{1}{6} [1 + (4)(3) + 5] = 2.33
\]

\[
\sigma_A = \frac{1}{6} (5-1) = 0.67
\]

The following table is generated in the same manner,

<table>
<thead>
<tr>
<th>Activity</th>
<th>( \bar{t}_{\text{mean}} )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>A</td>
<td>2.33</td>
<td>0.67</td>
</tr>
<tr>
<td>B</td>
<td>10.5</td>
<td>2.17</td>
</tr>
<tr>
<td>C</td>
<td>11.83</td>
<td>2.17</td>
</tr>
<tr>
<td>D</td>
<td>4.17</td>
<td>0.83</td>
</tr>
<tr>
<td>Finish</td>
<td>28.83</td>
<td>0</td>
</tr>
</tbody>
</table>

By observation, the critical path is Start-A-D-C-Finish; the project variance is

\[
\sigma^2 = (6)^2 + (2.17)^2 + (2.17)^2 + (18)^2 = 10.56
\]

The project standard deviation is

\[
\sigma = \sqrt{10.56} = 3.25
\]

So, we assume the completion times are normally distributed with a mean of 28.83 and a standard deviation of 3.25.

The standard normal variable is

\[
Z = \frac{28.83 - 15.42}{3.25} = 0.56
\]

Area under tail for \( Z = 0.56 \) is \( 0.2123 \)

So, \( 0.5 - 0.2123 = 0.2877 \) (28.77%)

\[
\boxed{\text{Area} = 0.2877}
\]

\[\lambda = 20\]

From page 1-22a)

(a) \( P[x < 17] = F(17) = \frac{e^{-20}(20)^{17}}{17!} = 0.076 \)

(b) \( P[x \leq 23] = F(0) + F(1) + F(2) + F(3) \)

\[
= \frac{e^{-20}(20)^{0}}{0!} + \frac{e^{-20}(20)^{1}}{1!} + \frac{e^{-20}(20)^{2}}{2!} + \frac{e^{-20}(20)^{3}}{3!}
\]

\[
= 2 \times 10^{-9} + 4.12 \times 10^{-8} + 4.12 \times 10^{-7} + 2.75 \times 10^{-6}
\]

\[
= 3.2 \times 10^{-6}
\]

From page 1-22b)

\[\mu = 1/23 \]

\[P[x > 25] = 1 - F(25) = e^{-25/23} = 0.337\]
16 FIRST, PLOT THE DATA TO SEE IF IT IS LINEAR

\[ y = \# \text{ OF VEHICLES} \]

\[ X = \# \text{ CARS IN TRAIN} \]

IT DOESN'T LOOK LINEAR, SO TRY THE FORM

\[ y = a + bx^2 \]

WHERE \( z = \log_{10} x \)

\[ \begin{array}{c|c|c}
    z & y & \sqrt{y} \\
    \hline
    0.3 & 14.8 & 3.87 \\
    0.7 & 18.0 & 4.22 \\
    1.0 & 23.0 & 4.78 \\
    1.3 & 29.9 & 5.47 \\
\end{array} \]

THAT ISN'T LINEAR EITHER, WE'VE OVERCOMPUTED THE CURVE, TRY THE FORM

\[ y = a + bx \]

WHERE \( w = z^2 = (\log_{10} x)^2 \)

\[ \begin{array}{c|c|c}
    w & y & \sqrt{y} \\
    \hline
    0.09 & 14.8 & 3.87 \\
    0.49 & 18.0 & 4.22 \\
    1.17 & 23.0 & 4.78 \\
    2.09 & 29.9 & 5.47 \\
\end{array} \]

FROM PAGE 1-13

\[ \sum w = 4.6 \quad \quad \sum y = 106.1 \]

\[ (\sum w)^2 = 21.16 \quad (\sum y)^2 = 11257.2 \]

\[ \sum w^2 = 6.43 \quad \sum y^2 = 23823.2 \]

\[ w = 4.6 \quad y = 106.1 \]

\[ \sum wy = 119.58 \]

\[ m = \frac{(5)(119.58) - (4.6)(106.1)}{(5)(4.43) - 21.16} = \frac{54.84}{10.99} = 5.0 \]

\[ b = 21.22 - (7.72)(9.2) = 14.12 \]

\[ y = 14.12 + 7.72w \]

\[ = 14.12 + 7.72z^2 \]

\[ = 14.12 + 7.72(\log_{10} x)^2 \]

The correlation coefficient is

\[ r = \frac{(5)(119.58) - (4.6)(106.1)}{\sqrt{(5)(4.43) - 21.16)(5)(23823.2) - 11257.2} = 0.999 \]

17 (a) USE THE 'CHARACTERISTIC EQUATION' METHOD TO SOLVE THE HOMOGENEOUS CASE (IT IS MUCH QUICKER TO USE LAPLACE TRANSFORMS, HOWEVER)

\[ x'' + 2x' + 2x = 0 \]

DIFF. EQ.

\[ R^2 + 2R + 2 = 0 \]

CHARACTERISTIC EQ.

COMPLETE THE SQUARE TO FIND \( R \)

\[ R^2 + 2R = -2 \]

\[ (R + 1)^2 = -2 + 1 \]

\[ R = -1 \pm i \]

\[ X(t) = A_1 e^{-t} \cos t + A_2 e^{-t} \sin t \]

Now use the initial conditions to find \( A_1 \) and \( A_2 \).

\[ x(0) = 0 \]

\[ 0 = A_1(1)(0) + A_2(1)(0) \]

\[ A_1 = 0 \]

Differential of the solution

\[ x(t) = A_2 \left[ e^{-t} \cos t - e^{-t} \sin t \right] \]

Using \( x'(0) = 1 \)

\[ 1 = A_2 \left[ (1)(1) - (0)(0) \right] \]

\[ A_2 = 1 \]

And the solution is \( x(t) = e^{-t} \sin t \)

(b) WITH NO DAMPING, THE DIFFERENTIAL EQUATION WOULD BE

\[ x'' + 2x = 0 \]

This has a solution of \( x = \sin \sqrt{2}t \), so \( \omega_n = \sqrt{2} \)

(c) \( x(t) = e^{-t} \sin t \)

\[ x'(t) = \left( e^{-t} \sin t + e^{-t} \cos t \right) \]

\[ = e^{-t} \sin t + e^{-t} \cos t \]

For \( x \) to be maximum, \( x'(t) = 0 \). Since \( e^{-t} \) is never large and \( e^{i} \) is very large, \( \cos t - \sin t \) must be zero. This occurs at \( t = \frac{\pi}{2} = 1.5708 \) radians, so

\[ x(1.5708) = e^{-1.5708} \sin (1.5708) = 0.322 \]

(d) USE THE LAPLACE TRANSFORM METHOD

\[ x'' + 2x' + 2x = \sin t \]

\[ s^2 X(s) + 2sX(s) + 2X(s) = \frac{1}{s^2 + 1} \]

\[ s^2 X(s) + 1 + 2sX(s) + 2X(s) = \frac{1}{s^2 + 1} \]

\[ X(s) \left[ s^2 + 2s + 2 \right] - 1 = \frac{1}{s^2 + 1} \]