Mathematics

WARM-UPS

1 $\sum_{J=1}^{5}(J+1)^2-1 = (1+1)^2-1 + (2+1)^2-1 + (3+1)^2-1 + (4+1)^2-1 + (5+1)^2-1$

$= 2^2+3^2+4^2+5^2+6^2-5 = 85$

2 THE ACTUAL VALUE IS

$y(2.7) = 3(2.7)^{.93}+4.2 = 11.756$

$y(2) = 3(2)^{.93}+4.2 = 9.916$

$y(3) = 3(3)^{.93}+4.2 = 12.534$

THE ESTIMATED VALUE IS

$9.916 + .7(12.534 - 9.916) = 11.749$

THE ERROR IS

$\frac{11.756 - 11.749}{11.756} = .0006$ OR $.06\%$

3 LET d BE THE DIAMETER

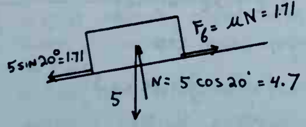
$V_{SPHERE} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = .524d^3$

$V_{CONE} = \frac{\pi}{3}r^2h = \frac{\pi}{3}\left(\frac{d}{2}\right)^2 h = .262d^2h$

BUT $.524d^3 = .262d^2h$

$h = 2.00d$

4



5 EXPAND BY 2^{ND} COLUMN

$-2\begin{vmatrix} 4 & 3 \\ 9 & 5 \end{vmatrix} = -2(20-27) = 14$

6 FROM EQN 6.3

$250° + 460° = 710°R$

$\frac{5}{9}(250-32) = 121.1°C$

7 FROM PAGE 1-42,

$K = 1.71\ EE{-}9\ \frac{BTU}{FT^2\text{-}HR\text{-}°R^4}$

$$\frac{\left(1.71\ EE{-}9\ \frac{BTU}{FT^2\text{-}HR\text{-}°R^4}\right)\left(17.57\ \frac{WATT\text{-}MIN}{BTU}\right)\left(\frac{1}{60}\ \frac{HR}{MIN}\right)}{\left(0.3048\ \frac{m}{ft}\right)^2\left(\frac{5}{9}\ K/°R\right)^4}$$

$= 5.66\ EE{-}8\ \frac{WATTS}{M^2\text{-}K^4}$

8 $y = 6 + .75(2-6) = 3.0$

9 THE SLOPE IS $\frac{9.5-3.4}{8.3-1.7} = .924$

USING THE FIRST POINT,

$(y-3.4) = .924(x-1.7)$

10 LET X BE THE NUMBER OF ELAPSED PERIODS OF .1 SECOND. LET Y_X BE THE AMOUNT PRESENT AFTER X PERIODS

$Y_1 = 1.001\ Y_0$

$Y_2 = (1.001)^2 Y_0$

$Y_N = (1.001)^N Y_0$

NOW $\frac{Y_X}{Y_0} = 2 = (1.001)^N$

$LOG(2) = N\ LOG(1.001)$

$N = 693.5$ PERIODS

$t = 69.35$ SECONDS

CONCENTRATES

1 FIRST, REARRANGE

$$\begin{aligned} X + Y \quad\quad &= -4 \\ X \quad\quad + Z &= 1 \\ 3X - Y + 2Z &= 4 \end{aligned}$$

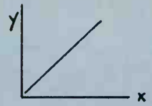
NOW, USE CRAMER'S RULE (PAGE 1-6)

$\begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 3 & -1 & 2 \end{vmatrix} = 1\begin{vmatrix} 0 & 1 \\ -1 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix} = (0+1) - (2-3) = 1+1 = 2$

$\begin{vmatrix} -4 & 1 & 0 \\ 1 & 0 & 1 \\ 4 & -1 & 2 \end{vmatrix} = -2 \quad \begin{vmatrix} 1 & -4 & 0 \\ 1 & 1 & 1 \\ 3 & 4 & 2 \end{vmatrix} = -6 \quad \begin{vmatrix} 1 & 1 & -4 \\ 1 & 0 & 1 \\ 3 & -1 & 4 \end{vmatrix} = 4$

$x^* = \frac{-2}{2} = -1 \quad y^* = \frac{-6}{2} = -3 \quad z^* = \frac{4}{2} = 2$

2 ALWAYS GRAPH THE DATA FIRST TO SEE IF IT IS A STRAIGHT LINE. IN THIS CASE, IT IS.



USE LINEAR REGRESSION (PAGE 1-13)

$N = 7$

$\Sigma x = 12550 \qquad \Sigma y = 12300$

$\bar{x} = 1792.9 \qquad \bar{y} = 1757.1$

$\Sigma x^2 = 3.117 \text{ EE7} \qquad \Sigma y^2 = 3.017 \text{ EE7}$

$(\Sigma x)^2 = 1.575 \text{ EE8} \qquad (\Sigma y)^2 = 1.513 \text{ EE8}$

$\Sigma xy = 3.067 \text{ EE7}$

$$m = \frac{7(3.067 \text{ EE7}) - (12550)(12300)}{7(3.117 \text{ EE7}) - (12550)^2} = .994$$

$$b = 1757.1 - .994(1792.9) = -25.0$$

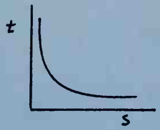
so $y = .994x - 25.0$

THE CORRELATION COEFFICIENT IS

$$r = \frac{7(3.067 \text{ EE7}) - (12550)(12300)}{\sqrt{[(7)(2.117 \text{ EE7}) - (12550)^2][(7)(3.017 \text{ EE7}) - (12300)^2]}}$$

$$= \sim 1.00$$

3 UPON GRAPHING THE DATA, WE SEE THAT IT IS NOT A STRAIGHT LINE



IT LOOKS LIKE AN EXPONENTIAL WITH FORM

$$t = b e^{ms}$$

OR PERHAPS

$$\log t = b + ms$$

TRY MAKING THE VARIABLE TRANSFORMATION

$R = \log t$

S	R
20	1.633
18	2.149
16	2.585
14	3.041

$N = 4$

$\Sigma S = 68 \qquad \Sigma R = 9.408$

$\bar{S} = 17 \qquad \bar{R} = 2.352$

$\Sigma S^2 = 1176 \qquad \Sigma R^2 = 23.215$

$(\Sigma S)^2 = 4624 \qquad (\Sigma R)^2 = 88.51$

$\Sigma SR = 155.28$

$$m = \frac{4(155.28) - (68)(9.408)}{4(1176) - (68)^2} = -.2328$$

$$b = 2.352 + .2328(17) = 6.3096$$

SO $R = 6.3096 - .2328S$

OR $\log t = 6.3096 - .2328S$

4 THIS IS A FIRST-ORDER LINEAR DIFFERENTIAL EQUATION. (PAGE 1-31)

$$\mu = \exp\left[\int -1\, dx\right] = e^{-x}$$

$$y = e^{x}\left[2\int e^{-x} x e^{2x} dx + c\right]$$

$$= e^{x}\left[2xe^{x} - 2e^{x} + c\right]$$

BUT $y = 1$ WHEN $x = 0$

$$1 = 1[0 - 2 + c] \quad \text{or } c = 3$$

so $y = 2e^{2x}(x-1) + 3e^{x}$

5 SOLVE THE CHARACTERISTIC QUADRATIC EQUATION:

$$R^2 - 4R - 12 = 0$$

$$R = 6, -2$$

so $y = a_1 e^{6x} + a_2 e^{-2x}$

6 LET X_t = POUNDS OF SALT IN TANK AT TIME t

$X_0 = 60$

X' = RATE AT WHICH SALT CONTENT CHANGES

2 = POUNDS OF SALT ENTERING EACH MINUTE

3 = GALLONS LEAVING EACH MINUTE.

THE SALT LEAVING EACH MINUTE IS

$$3\left(\begin{matrix}\text{CONCENTRATION}\\ \text{IN LB/GAL}\end{matrix}\right) = 3\left(\frac{\text{SALT CONTENT}}{\text{VOLUME}}\right) = 3\left(\frac{X}{100-t}\right)$$

$$X' = 2 - 3\left(\frac{X}{100-t}\right)$$

OR

$$X' + \frac{3X}{100-t} = 2$$

THIS IS FIRST ORDER LINEAR (PAGE 1-31)

$$u = \exp\left[3\int\frac{dt}{100-t}\right] = (100-t)^{-3}$$

$$X = (100-t)^3\left[2\int\frac{dt}{(100-t)^3} + K\right]$$

$$= 100 - t + K(100-t)^3$$

BUT $X = 60$ AT $t = 0$

SO $K = -.00004$

$$X = 100 - t - .00004(100-t)^3$$

$$X_{60} = 37.44 \text{ POUNDS}$$

7 IF C IS POSITIVE, THEN $N(\infty) = \infty$, WHICH IS CONTRARY TO THE GIVEN DATA. SO $C \le 0$.

IF $C = 0$, THEN $N(\infty) = \frac{a}{1+b} = 100$ WHICH IS POSSIBLE DEPENDING ON a, b

IF $C = 0$, THEN $N(0) = \frac{a}{1+b} = 10$, WHICH CONFLICTS WITH THE PREVIOUS STEP.

SO $C < 0$. THEN $N(\infty) = a$, SO $\underline{\underline{a = 100}}$

NOW $N(0) = \frac{a}{1+b} = \frac{100}{1+b} = 10$, SO $\underline{\underline{b = 9}}$

$$\frac{dN}{dt} = -100(1+9e^{ct})^{-2}(9)e^{ct}(c)$$

IF $t = 0$, THEN $C = \underline{\underline{-.0556}}$

8

$$\frac{dy}{dx} = 3x^2 - 18x$$

$3x^2 - 18x = 0$ AT ALL EXTREME POINTS

$x^2 - 6x = 0$ AT $x = 0$, $x = 6$

$$\frac{d^2y}{dx^2} = 6x - 18$$

$6x - 18 = 0$ AT INFLECTION POINTS

$x = 3$ IS AN INFLECTION POINT

$6(0) - 18 = -18$, SO $x = 0$ IS A MAXIMUM

$6(6) - 18 = 18$, SO $x = 6$ IS A MINIMUM

9 THE ENERGY CONTAINED IN ONE GRAM OF ANY SUBSTANCE IS

$$E = MC^2 = (.001)\text{ KG }(3\text{ EE}8)^2(\text{M/S})^2$$

$$= 9\text{ EE}13 \text{ JOULES}$$

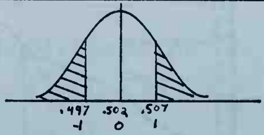
$$(9\text{ EE}13)\text{ J}\left(\frac{1}{1000}\right)\frac{\text{KJ}}{\text{J}}(.9478)\frac{\text{BTU}}{\text{KJ}}$$

$$= 8.53\text{ EE}10 \text{ BTU}$$

$$\#\text{ TONS} = \frac{8.53\text{ EE}10\text{ BTU}}{(13{,}000)\frac{\text{BTU}}{\text{LB}}(2000)\frac{\text{LB}}{\text{TON}}}$$

$$= 3281 \text{ TONS}$$

10



THE STANDARD NORMAL VARIABLES ARE

$$z_1 = \frac{.502 - .497}{.005} = 1$$

$$z_2 = \frac{.507 - .502}{.005} = 1$$

a) $p\{\text{DEFECTIVE}\} = 2[.5 - .3413] = .3174$

b) $p\{3,2\} = \frac{3!}{(3-2)!\,2!}(.3174)^2(1-.3174)^1$

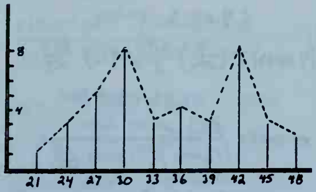
$$= .2063$$

c) $(8)(200)(.3174) = 507.8$

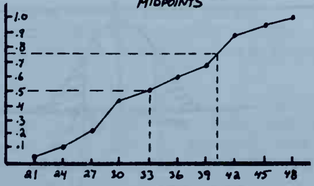
11

THE RANGE OF SPEEDS IS $(48-20)=28$. SINCE THERE ARE NOT A LOT OF OBSERVATIONS, 10 CELLS WOULD BE BEST. CHOOSE THE CELL WIDTH AS $\left(\frac{28}{10}\right) \approx 3$

INTERVAL	MID-POINT	FREQ.	CUM FREQ.	CUM %
20-22	21	1	1	.03
23-25	24	3	4	.10
26-28	27	5	9	.23
29-31	30	8	17	.43
32-34	33	3	20	.50
35-37	36	4	24	.60
38-40	39	3	27	.68
41-43	42	8	35	.88
44-46	45	3	38	.95
47-49	48	2	40	1.00



MIDPOINTS



MIDPOINTS

f) USE THE CUMULATIVE DISTRIBUTION GRAPH. FOR 75% (.75), THE CELL MID-POINT IS APPROXIMATELY 40

g) USE THE CUMULATIVE GRAPH TO FIND THE MID-POINT FOR 50%. THIS OCCURS AT APPROXIMATELY 33.

$\Sigma x_i = 1390$, SO MEAN $= \frac{1390}{40} = 34.75$

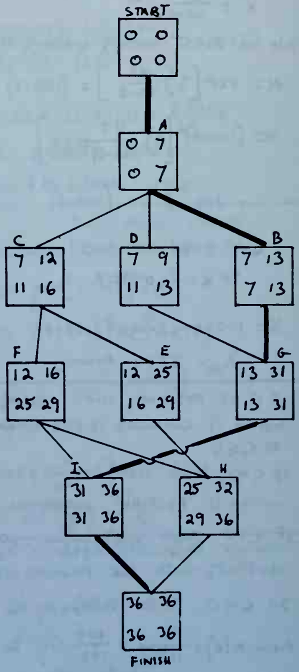
h) $\Sigma x^2 = 50496$

$$\sigma = \sqrt{\left(\frac{50496}{40}\right) - \left(\frac{1390}{40}\right)^2} = 7.405$$

i) $s = \sqrt{\frac{N}{N-1}}(\sigma) = \sqrt{\frac{40}{39}}(7.405) = 7.500$

j) $s^2 = 56.24$

12 NO CONTRACT DEADLINE WAS GIVEN, SO ASSUME 36 AS A SCHEDULED TIME. LOOK FOR A PATH WHERE $(LS-ES)=0$ EVERYWHERE



c) 36
d) 36
e) 0
f) FLOAT IS SAME AS SLACK $=0$

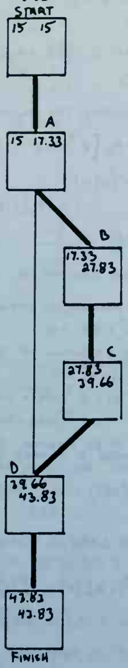
13 TO SOLVE THIS AS A REGULAR CPM PROBLEM, IT IS NECESSARY TO CALCULATE t_{mean} AND σ FOR EACH ACTIVITY. FOR ACTIVITY A,

$$t_{mean} = \frac{1}{6}\left[1 + (4)(2) + 5\right] = 2.33$$

$$\sigma_A = \frac{1}{6}(5-1) = .67$$

THE FOLLOWING TABLE IS GENERATED IN THE SAME MANNER.

ACTIVITY	t_{mean}	σ
START	0	0
A	2.33	.67
B	10.5	2.17
C	11.83	2.17
D	4.17	.83
FINISH	0	0
	28.83	



BY OBSERVATION, THE CRITICAL PATH IS START-A-B-C-D-FINISH, THE PROJECT VARIANCE IS

$$\sigma^2 = (.67)^2 + (2.17)^2 + (2.17)^2 + (.83)^2 = 10.56$$

THE PROJECT STANDARD DEVIATION IS

$$\sigma = \sqrt{10.56} = 3.25$$

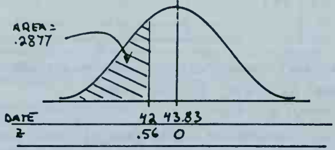
SO, WE ASSUME THE COMPLETION TIMES ARE NORMALLY DISTRIBUTED WITH A MEAN OF 28.83 AND A STANDARD DEVIATION OF 3.25

THE STANDARD NORMAL VARIABLE IS

$$z = \frac{28.83 + 15 - 42}{3.25} = .56$$

AREA UNDER TAIL FOR $z = .56$ IS .2123

SO - $.5 - .2123 = .2877$ (28.77%)



14 $\lambda = 20$

FROM PAGE 1-22,

(a) $p\{x=17\} = f(17) = \dfrac{e^{-20}(20)^{17}}{17!} = .076$

(b) $p\{x \le 3\} = f(0) + f(1) + f(2) + f(3)$

$$= \frac{e^{-20}(20)^0}{0!} + \frac{e^{-20}(20)^1}{1!} + \frac{e^{-20}(20)^2}{2!} + \frac{e^{-20}(20)^3}{3!}$$

$= 2$ EE-9 $+ 4.12$ EE-8 $+ 4.12$ EE-7 $+ 2.75$ EE-6

$= 3.2$ EE-6

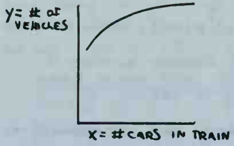
15 FROM PAGE 1-22,

$\mu = 1/23$

$$p\{x > 25\} = 1 - F(25) = e^{-\left(\frac{1}{23}\right)(25)}$$

$$= .337$$

16 FIRST, PLOT THE DATA TO SEE IF IT IS LINEAR

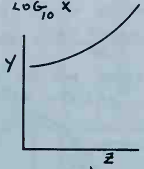


IT DOESN'T LOOK LINEAR, SO TRY THE FORM

$y = a + bz$

WHERE $z = \log_{10} x$

z	y
.3	14.8
.7	18.0
.9	20.4
1.08	23.0
1.43	29.9

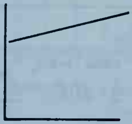


THAT ISN'T LINEAR EITHER. WE'VE 'OVERCOMPENSATED' THE CURVE. TRY THE FORM

$y = a + bw$

WHERE $w = z^2 = (\log_{10} x)^2$

w	y
.09	14.8
.49	18.0
.81	20.4
1.17	23.0
2.04	29.9



FROM PAGE 1-13

$\Sigma w_i = 4.6$ $\qquad \Sigma y_i = 106.1$

$(\Sigma w_i)^2 = 21.16$ $\qquad (\Sigma y_i)^2 = 11257.2$

$\Sigma w_i^2 = 6.43$ $\qquad \Sigma y_i^2 = 2382.2$

$\bar{w} = \frac{4.6}{5} = .92$ $\qquad \bar{y} = \frac{106.1}{5} = 21.22$

$\Sigma w_i y_i = 114.58$

$$m = \frac{(5)(114.58) - (4.6)(106.1)}{(5)(6.43) - 21.16} = \frac{84.84}{10.99}$$

$$= 7.72$$

$$b = 21.22 - (7.72)(.92) = 14.12$$

SO,

$$y = 14.12 + 7.72\,w$$
$$= 14.12 + 7.72\,z^2$$
$$= 14.12 + 7.72\,(\log_{10} x)^2$$

THE CORRELATION COEFFICIENT IS

$$r = \frac{(5)(114.58) - (4.6)(106.1)}{\sqrt{[(5)(6.43) - 21.16][(5)(2382.2) - 11257.2]}} \approx 0.999$$

{THE TRANSFORMATION $y = a + b\sqrt{x}$ YIELDS $y = 9.11 + 4\sqrt{x}$ AND $r = 0.999$, EQUALLY GOOD.}

17

(a) USE THE 'CHARACTERISTIC EQUATION' METHOD TO SOLVE THE HOMOGENEOUS CASE. (IT IS MUCH QUICKER TO USE LAPLACE TRANSFORMS, HOWEVER)

$x'' + 2x' + 2x = 0$ DIFF. EQ.

$R^2 + 2R + 2 = 0$ CHARACTERISTIC EQ.

COMPLETE THE SQUARE TO FIND R

$$R^2 + 2R = -2$$
$$(R+1)^2 = -2 + 1$$
$$R + 1 = \pm\sqrt{-1}$$
$$R = -1 \pm i$$

SO,

$$x(t) = A_1 e^{-t}\cos t + A_2 e^{-t}\sin t$$

NOW, USE THE INITIAL CONDITIONS TO FIND A_1 AND A_2.

$x(0) = 0$

$$0 = A_1(1)(1) + A_2(1)(0)$$

SO, $A_1 = 0$

DIFFERENTIATING THE SOLUTION,

$$x'(t) = A_2\left[e^{-x}\cos x - \sin x\, e^{-x}\right]$$

USING $x'(0) = 1$

$$1 = A_2[(1)(1) - (0)(1)]$$

SO, $A_2 = 1$

AND, THE SOLUTION IS $x(t) = e^{-t}\sin t$

(b) WITH NO DAMPING, THE DIFFERENTIAL EQUATION WOULD BE

$$x'' + 2x = 0$$

THIS HAS A SOLUTION OF $x = \sin\sqrt{2}\,t$, SO $\omega_{nat} = \sqrt{2}$

(c)
$$x(t) = e^{-t}\sin t$$
$$x'(t) = e^{-t}\cos t - \sin t\, e^{-t}$$
$$= e^{-t}(\cos t - \sin t)$$

FOR x TO BE MAXIMUM, $x'(t) = 0$. SINCE e^{-t} IS NOT ZERO UNLESS t IS VERY LARGE, $\cos t - \sin t$ MUST BE ZERO. THIS OCCURS AT $t = .785$ RADIANS, SO

$$x(.785) = e^{-.785}\sin(.785)$$
$$= .322$$

(d) USE THE LAPLACE TRANSFORM METHOD

$$x'' + 2x' + 2x = \sin(t)$$

$$\mathcal{L}(x'') + 2\mathcal{L}(x') + 2\mathcal{L}(x) = \mathcal{L}(\sin(t))$$

$$s^2\mathcal{L}(x) - 1 + 2s\mathcal{L}(x) + 2\mathcal{L}(x) = \frac{1}{s^2+1}$$

$$\mathcal{L}(x)\left[s^2 + 2s + 2\right] - 1 = \frac{1}{s^2+1}$$